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THE CRYSTAL FORMS OF METALLIC SELENIUM AND SOME OF THEIR PHYSICAL PROPERTIES.

By F. C. Brown.

WHETHER light-sensitive selenium is made up of homogeneous crystal structure or of complex units is of importance in explaining the nature of light action in selenium. Thus the author has been led into a study of the crystal forms of metallic selenium, which study has been particularly fruitful in that new crystal forms of very large size have been isolated. These crystals are of such a size that the optical and photoelectric properties of individual crystals can be studied. Thus we are able to eliminate some of possible causes of complexity in selenium, such as polymorphic crystal mixtures, unknown condition of crystal contacts, irregular placing of the crystals, and impurities. The fact that these new crystal forms are light-sensitive opens up a large field of investigation of which this paper is only preliminary.

Muthman[1] described only one crystal form of metallic selenium of the hexagonal rhombohedral system. These crystals were produced by the sublimation of the vapor of selenium in air and were never larger than 0.2×0.5 mm.

Saunders[2] also describes only one form. His conclusions were based upon the changes in volume accompanying temperature changes, as determined by dilatometric measurements. The difficulty with Saunders's method is that different crystal forms having identical or only slightly varying density could not be detected.

Marc[3] following the experiments of Uljanin[4] who observed two distinct crystallographic structures under the microscope, and following his own

[1] Zeit. f. Kryst., 17, 356, 1890.

[2] "The Allotropic Forms of Selenium," Journ. of Phys. Chem., Vol. 4, p. 423, 1900.

[3] Die Physikalische Chemischen Eigenschaften des Metallisches Selens, Verlag von Voss, 1909.

[4] Wied. Ann., 34, p. 241, 1888.

experiments on selenium cells, was led to believe in the existence of several forms of metallic selenium, which through the action of light underwent polymorphic transformations. By the investigations of the heat changes, the electrical conductivity and a microscopic investigation of the surface he was able to detect with certainty the existence of two polymorphic forms. The one form was crystallized by heating the selenium at lower temperatures in the neighborhood of 100°. It was essentially non conducting and was designated form *A*. At temperatures of 205° to 215° this selenium went over into selenium *B*, which was found to be quite conducting. A microscopic investigation showed the form *B* to be of uniformly round mounds, while that crystallized at 200° contained long crystals. The crystals observed however were only of the size of about 0.004 mm. Marc's results which led him to believe in the existence of two forms *A* and *B* and which led him to believe in the dynamic equilibrium of these forms, were not sufficient to convince him that there could not be other forms. One of the phases producing equilibrium he regarded as a vapor phase, thus giving only one solid form for two phases.

Recently White[1] did some very excellent work with selenium blocks crystallized at 200°. He found the interesting result that a greater change of conductivity was produced when the incident light was in the direction of flow of the current than when the illumination and electric current directions were at right angles to each other. He concluded that in the selenium block the highest resistance is at the electrode contacts and also that the greatest change of conductivity by illumination takes place at the electrodes. However in interpreting his results he did not consider the change of conductivity arising from increased potential gradient when the blocks were illuminated at the contact surfaces. He also observed a number of phenomena such as has been observed in the so-called selenium cells.[2] This work of White further suggests the advisability of obtaining single isolated crystals of selenium in which the physical conditions, may be known and controlled.

THE PRODUCTION OF ISOLATED CRYSTALS.

The new crystal forms that are the subject of discussion were in all cases produced by the sublimation of the vapor, either in a high vacuum or at atmospheric pressure. The vitreous selenium which it was desired to transform into crystals was placed in one end of a glass tube of 35 mm. inside diameter and of 30 to 60 cm. length. This glass tube fitted snugly

[1] Phil. Mag., Ser. VI., Vol. 27, p. 370, 1914.

[2] Unquestionably the term "selenium bridge" as he suggests is a better term than "selenium cells."

into a cylindrical electrical oven of 30 cm. length, of which one end was closed. Near the open end of the oven there was a rather large temperature gradient whether the tube was evacuated or not. All the selenium that deposited in any form was usually within a distance of less than 7 cm. along the tube. A current of about six amperes in the oven was sufficient to heat the selenium to be transformed to about 270°. The selenium was kept at this temperature for a period varying between one day and one week. But there was no temperature regulator and consequently there was considerable variation in the temperature during the day, with either the dynamo or the storage battery as a source of current. The selenium was allowed to sublime on the walls of the glass tube, on a thermometer bulb, or on a form with wire electrodes for a light-sensitive selenium bridge. It was hoped that the thermometer would register the temperature at which the crystals formed on it, but the temperature gradient was so large that no reliance was placed in the readings.

The result may be stated in a general way as follows. The largest crystals in every case were formed at the highest temperature at which the selenium sublimed in the closed tube. This temperature is believed to be above 210°. It should have been ascertained more accurately, but the effort was rather discouraging because of the absence of automatic temperature control. These crystals became smaller as the cooler portion of the tube was approached, until about 3 cm. back of the largest crystals there was a continuous sheath of selenium of a silver luster surrounding the inner wall of the tube. On some occasions the hotter edge of this sheath was fringed with small well-developed rhombohedral-hexagonal crystals (see Fig. 1) such as described by Muthman[1]. Back of this silver-like sheath was sometimes discerned a rather black sheath which also had a metallic luster. Further along the tube at temperatures perhaps below 100° was usually a very thin deposit of red amorphous selenium. I was rather surprised to find so small an amount of selenium deposited in the red amorphous form, and so large a quantity in the metallic forms. I was also surprised in some cases to find so large a fraction of the selenium as one fifth of the total to be deposited in the form of large crystals. This is rather a sufficient argument against any view that might presuppose impurities to form the large crystals. The selenium that was placed in the tube to be sublimated was Merck's purified selenium.

Invariably the largest crystals were formed at the highest temperature, but it is not clear what determined the form they should take. The photograph in Fig. 2 shows distinctly varying formations that occurred

[1] Loc. cit.

in two separate tubes. The acicular crystals were sometimes 11 mm. long and not more than 0.2 mm. in any other dimension. Usually they developed a number together in a cactus-like growth, with from 3 to 20 spines starting from a common center. These spines or needles were quite stiff and tough, rough handling and even dropping very rarely injuring them. The needles of any given cluster were usually very much alike in size and appearance. They always had a metallic luster by reflected light. If fully developed there were six surfaces. And yet some entire clusters apparently developed with any number of surfaces less than six. Under the microscope these crystals seemed the least transparent of any that were produced.

The crystals of the second type shown in Fig. 2 were large flat crystals with parallel surfaces usually less than 0.3 mm. apart. The length of one of these crystals indicated in the photograph was 9 mm., and for the greater part of the length the width was about 2 mm. These crystals frequently began growth by the addition of longer and longer crystals to the side of the last crystal laid down. This accounts for the triangular point frequently seen on these large crystals. Sometimes these flat crystals were nearly square and again diamond shaped. Frequently the flat would develop in places as a rolling surface and also be distorted at the edges. But in any given run with fixed resistance coils in series with the oven heater, there was an unmistakable resemblance between all the crystals formed in a given vicinity of the tube. The largest crystals of this form appeared in a tube in which the air pressure had been reduced to less than 0.01 mm. previous to the sealing off of the tube.

The crystal groups to the left in Fig. 3 are also typical of those that were formed in one tube. They were as long as 4 mm. and as wide as 0.8 mm. They were the result of uniform twinning in such a manner that an open or closed V-shaped gauge was formed by each spine.

Yet another interesting formation that appeared once was a large number of very flexible sheets, sometimes 4 mm. long and as wide as .6 mm. They developed largely one in a place. As they waved about freely by slight air currents, they much resembled a miniature aluminium leaf as used in an electroscope. The thickness of one of these was of the order of 0.01 mm. The other forms described above showed no such flexibility at all. The relative dimensions of two of these crystals may be estimated from the reproduced photograph in Fig. 5. The length of the longest is about 3 mm.

The crystals that formed at the edge of the metallic sheath were only occasionally like those described by Muthman. Fig. 3 shows a photograph of a much larger formation, some as large as 4 mm. which appeared

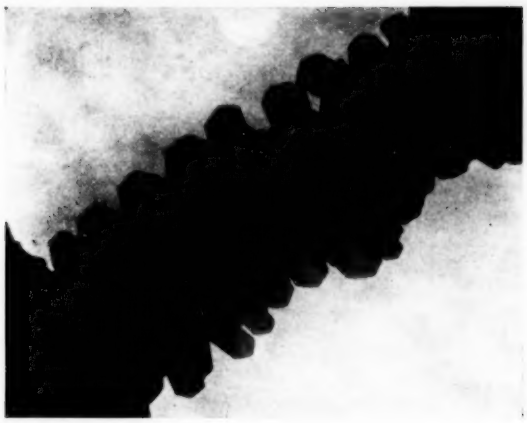

Fig. 1.

Magnified hexagonal crystals, natural size of single crystals not over 0.1 mm. in width.

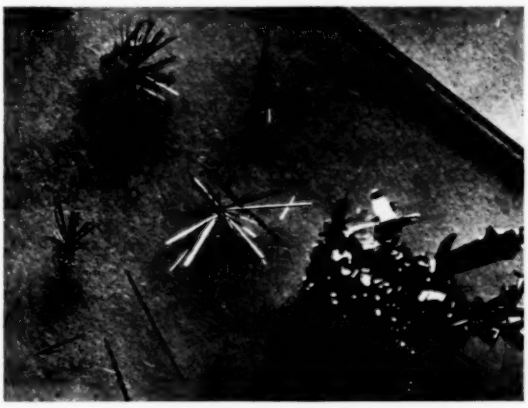

Fig. 2.

(*a*) Groups of acicular crystals to the left. Natural size of longest ones about 11 mm. (*b*) Growth of lamellar crystals in lower right-hand corner, the largest of which as seen in the figure was about 9.0×2×0.4 mm.

Fig. 3.

The lower part of the photograph shows the growth of single and twinned crystals of maximum length about 4 mm. at the edge of a crystal mass of finer grain. The scattered crystals are of the same formation.

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Fig. 4.

The above crystals are the same form as those shown in Fig. 3, but were photographed by transmitted light with a magnification of about 100 times.

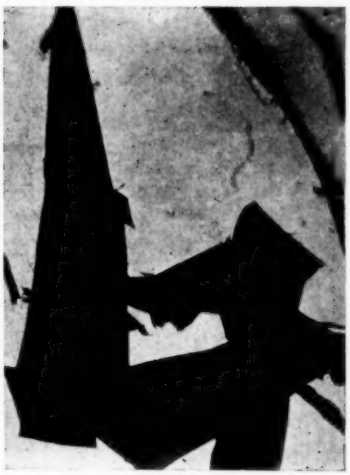

Fig. 6.

Lamellar crystals of gradually increasing width. Magnification about 100. Note the striations making an angle of about 35° with the edge.

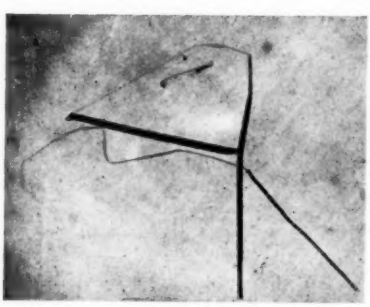

Fig. 5.

Two crystals of flexible lamellar type. The longest of the above was about 4 mm. long. The width and thickness may be estimated from the reproduction.

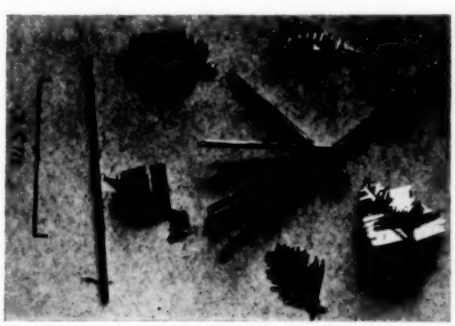

Fig. 9.

Crystals of metallic selenium by sublimation. Acicular hexagonal and acicular with side branches. Side branches perpendicular to stem are hexagonal. Those leaving at angle of 60° are lamellar plates showing parallel extinction.

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under similar circumstances. Some of the same crystals were magnified much more and photographed by transmitted light as seen in Fig. 4.

The manner in which some of the semi-flexible lamellæ developed is shown by the photograph (transmitted light) in Fig. 6. The striations can be seen running across the surface at about an angle of 35°. With the aid of polarized light these striations become much more marked. They indicate the way in which the large sheet lamella are sometimes built up.

Yet other crystal forms of small dimensions that appeared under the microscope are shown in Fig. 7. The largest of these was about .05 mm. All of these were observed many times. However they were too small for a study of their electrical properties.

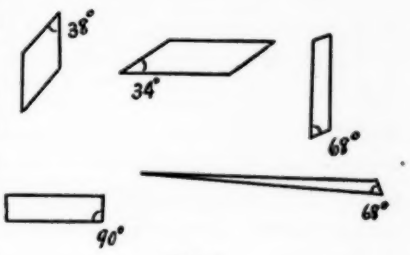

Fig. 7.

Unfortunately all the large crystal forms that I have described were formed at high temperatures. When we have obtained a satisfactory temperature regulator, we shall endeavor to obtain large-sized crystals at about 150°. No doubt more constant and better defined temperatures will produce more regular and larger crystals of the types that I have described, and perhaps new ones.

It is probable that all the crystals belong to the third crystal system, but before this can be decided definitely I believe that it will be at least advisable to obtain more large crystal forms, in order to determine accurately the angles between the faces.

Naturally the question arises as to why selenium crystallizes in so many forms at so nearly the same temperatures. I wish to proclaim a most profound ignorance as to this question. However there is strong indication that the partial pressure of the vapor of selenium and the temperature gradient in the tube at the place of formation are the prime factors. For example I have noted repeatedly that the thin lamellar form appeared when the oven temperature was low, where the amorphous selenium was placed for sublimation. Without doubt the conditions for formation of the crystals should be studied more carefully.

A photograph of some interesting crystals formed at atmospheric pressure is shown in Fig. 9.

THE PROPERTY OF DOUBLE REFRACTION.

All the forms of metallic selenium that have thus far been observed display a distinctly metallic appearance by reflected light. The surfaces

were nearly always quite plane and consequently, as the photographs show, the crystals appeared very light or very dark depending on the position of the surface angle with regard to the direction of maximum illumination. It is to be observed in Figs. 2 and 3 that the diffuse light is much less than that from the white paper which served as a background. Even the fine-grained crystal masses formed at lower temperatures and shown in Fig. 3 show a distinct black with contrasting white specks here and there. Likewise all the crystal forms except the acicular ones transmit light even through great thicknesses, and it is interesting to note that wherever the crystal transmits light it displays the property of double refraction. When examined under crossed nicols the transmitted light is most frequently some tinge of deep red, but various formations and thicknesses of crystal show the transmitted light to be distinctly a blue, yellow, orange, green or even white. The fully developed rhombohedral hexagonal crystals in Fig. 1 show a deep ox-blood red under crossed nicols. The transmitted light seems to appear as a body color in the crystal. This may be due to a deviation or scattering of the light inside the crystal. In a lamellar crystal such as shown in Fig. 6, several variations of color are observable at different places. The striations become particularly distinct under crossed nicols by the variation of color and color density.

The elementary crystal forms and the position in which they show extinction of light under crossed nicols are shown approximately correct in Fig. 8. The nicol prisms are placed as shown by the arrows. The dark positions are those of light extinction and the light positions those in which the light was readily transmitted, in some color or other. The crystal *a* is for one having acute angles of about 35°. Crystal *c* is obviously the multiplication of *a*. Crystal *b* is one having 68° angles. Its axis obviously runs lengthwise of the crystals. In *d* is a distinctly right-angled lamellar form, also with optic axis running lengthwise of the crystal. The crystal *e* which is also representative of a large number that were observed, showed the optic axis to run parallel to the very short edge. The acute angle here was about 68°. Thus this crystal may be regarded as built up of a large number of *b* crystals of gradually increasing length. It happens that all the crystals represented in Fig. 8 were

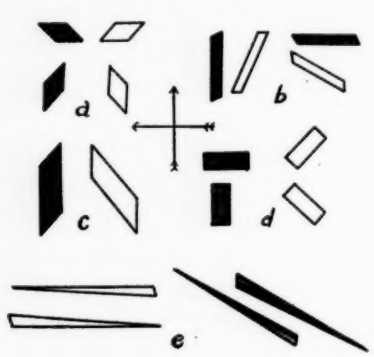

Fig. 8.

drawn under the same magnification, but they might have been chosen in widely varying sizes. It appears contrary to first expectation that the crystal in Fig. 6 for example is not built up of units of *e*, but rather of units of the crystal *b*. Under crossed nicols it shows extinction parallel to the striations. The crystal in Fig. 5 when viewed flatwise shows extinction parallel to the length of the crystal, indicating the optic axis to be lengthwise of the crystal. This long and flexible crystal may therefore be regarded as built up of the crystals of type *d*, in Fig. 8. The crystals in Fig. 1 show extinction parallel to the long crystallographic axis.

It may be of passing interest to deviate from the topic of this paper to note that the glittering red plates of the red crystalline variety of selenium show double refraction and also that they are not at all metallic in appearance, nor do they show electrical conductance even under very high pressures.

Some time ago the depth of penetration of light into selenium[1] was deduced to be greater than 0.014 mm. This result is very much larger than that of other observers. Pfund,[2] for example, found the depth to be about 10^{-6} cm. White[3] on the other hand believed his selenium blocks indicated a greater penetration than any result indicated above. It seems that the depth of penetration should and does vary with the size, character and positions of the crystals. I have observed a large amount of light transmitted through a lamellar crystal such as shown in Fig. 2 of thickness as great as 0.3 mm. This transmitted light was a deep red and was deviated from its path on emergence through a large angle, perhaps 40°. Thus in a mixture of small crystals adjacent to each other the path might be devious indeed. There is no reason to believe that the light or the light action might not travel several millimeters in a well-formed crystal. Evidence will be mentioned later which indicates that the total number of electrons liberated does not vary greatly whatever may be the deviations of the light.

THE EFFECT OF PRESSURE.

The pressure effect on selenium was discovered in 1905[4] showing a very large increase of conductivity for increased pressure up to 1,000 atmospheres. Montèn[5] extended the study independently up to 3,000 atmospheres and observed conductivity changes more than a hundred-

[1] PHYS. REV., 34, p. 201, 1912.
[2] PHYS. REV., 28, p. 324, 1909.
[3] Loc. cit.
[4] PHYS. REV., Vol. 20, p. 185, 1905.
[5] Arkiv för Matematik, Astronomi och Fysik, Bd. 4, No. 31, p. 1, 1908.

fold. About the same time a study of the light sensitiveness under varying hydraulic pressures[1] very decidedly indicated that the pressure produced a genuine change in the selenium and did not alter the contact resistances. This argument was based on the presumption that no part of the light action was of a nature of alteration of contact resistances. A recent consideration of the similarity of certain physical properties in light-sensitive selenium and crystal contacts[2] has led the author to intimate that the early presumption referred to above might be wrong. White's recent work[3] with selenium blocks showed the largest change of resistance by light to take place at the electrodes.

A study of a single crystal, in contact with plane electrodes, under varying pressure should reveal important information as to the seat of the pressure effect in ordinary light sensitive selenium as well as in the individual crystal. This pressure effect was easily studied by placing the crystal between two surfaces as shown in Fig. 10. The pressure was increased by merely adding weights. This pressure was applied in the

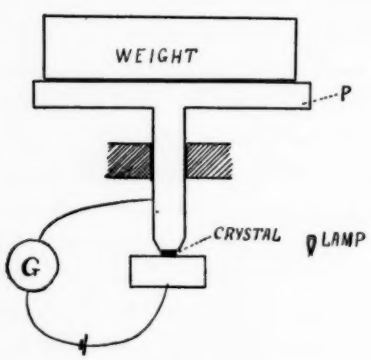

Fig. 10.

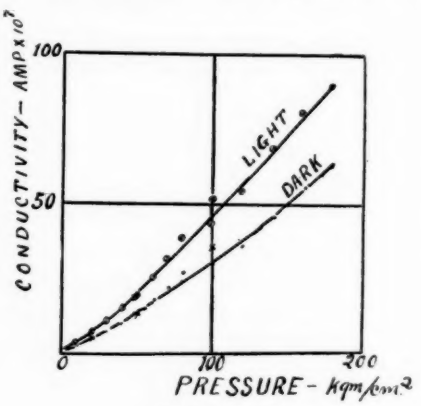

Fig. 11.

direction of the flow of current. The curves in Fig. 11 show the remarkable variation of the conductivity of a single lamellar crystal such as shown in Fig. 2 with change in pressure. The lower curve is the conductivity in the dark and the upper one is for the conductivity in the light. In the dark the conductivity increases about 120 times for an increase of pressure of 180 atmospheres. The crosses on the curves show the conductivity values both in the light and in the dark as the pressure was released. Thus it appears that the selenium crystal attains a given equilibrium under a given pressure. Under higher pressures the con-

[1] See paper by Brown and Stebbins, PHYS. REV., 26, p. 273, 1908.

[2] PHYS. REV., N. S., Vol. 1, p. 245, 1912, and Proc. Iowa Acad., 1913.

[3] Loc. cit.

ductivity was unsteady, so that thus far the action of higher pressures is uncertain. However I succeeded in increasing the conductivity of one crystal about a million times by pressure. Under these conditions it was not light sensitive. The fact that there was only a slight hysteresis as illustrated by the fact that the points on the curves shown were not taken in exactly regular order, is strong evidence that the pressure effect is in the selenium crystal and not at the electrode contacts.

THE VARIATION OF THE LIGHT-SENSITIVENESS WITH PRESSURE.

Experimental Results.—With the apparatus shown in Fig. 10 it was easy to determine the light-sensitiveness with different pressures. The interesting result followed that the light action increased with increase of pressure. In the following table is given the conductivity (c) in the dark, which prevailed under the varying pressures recorded in the upper curve of Fig. 11, and also the proportional change of conductivity, $\Delta C/C$, produced by constant illumination. The conductivity is indicated in

C.	$\Delta C/C$.	C	$\Delta C/C$.
.5	.6	21.6	.43
2.0	.5	28.5	.46
4.5	.45	35	.50
7.4	.42	46.5	.46
10	.44	55	.45
13	.42	62	.43
17.8	.40		

terms of divisions deflection of the galvanometer, G. It is observed that the conductivity in the dark varied by a factor of 124. Within this large range, the light-action as measured by the change of conductivity is directly proportional to the conductivity in the dark. It may be concluded for pressures up to 180 atmospheres, that the percentage increase of conductivity by a given illumination is constant. This result leads to a remarkable interpretation as to the nature of light action.

Interpretation of Result $\Delta C/C$ = Constant.—First it will be seen that the resistance exists in the selenium itself and not at electrode contacts. For suppose the light to produce a change of resistance ΔR in the selenium. Then the proportional decrease of resistance by the light, $\Delta R/(Rs + Rc)$, would get constantly greater inversely proportional to the change of resistance, where Rs and Rc are the resistances of the selenium and the contacts respectively. This result would follow whether the light action were at the contacts or in the selenium itself. For pressures of only a few grams the above ratio was sometimes observed to increase with increase of pressure, but with pressures of 50 gm. or more the ratio of $\Delta C/C$ (or $\Delta R/R$) was constant as shown in the preceding table. It is

concluded therefore that contact resistances are of very secondary or negligible consequence in connection with the ordinary light-electric phenomena in selenium crystals. This result is quite consistent with other experiments to be published later.

The constancy of the ratio of $\Delta C/C$ could be explained if it were possible that the increased amount of the conducting component produced by pressure were the only component that could still further be acted upon by light. But this seems so unreasonable, and it is certainly inconsistent with the results obtained by the author in his various works on selenium cells.

The only reasonable interpretation, it seems, must be worked out somewhat along the following line. Suppose a number of isolated conducting centers throughout the crystal and that this number is a function of the pressure. Generally such an insulated center would not conduct at all, but as the pressure becomes greater a large number of these centers would come within each other's confines, thus increasing the conductivity of the crystal. If the number of centers is small, the conductivity should increase as the number of centers. If a given illumination produces a definite number of new centers, then these new centers should bridge over gaps between old centers approximately proportional to the number of gaps filled, which should vary as the number of old centers or circuits. Of course if the selenium should have a very large number of gaps already completed without the light action, then the light would not have the opportunity of completing a circuit so frequently for every center acted upon. In agreement with this notion I found that when pressures, sufficient to increase the conductivity a million fold were applied that the light sensitiveness diminished to practically zero. It may be convenient to modify this explanation later to satisfy the electron theory of conduction. However it may be mentioned that as a result of further experiments with these crystals Dr. Sieg and myself are compelled to revise the electron theory of electrical conduction. These experiments will be described later.

THE ACTION OF LIGHT.

Essentially in the Body of the Crystal.—In view of White's[1] experiments with selenium blocks in which he observed the greatest change of conductivity at the electrode contacts, it seemed advisable to try the same experiment with a single crystal. This time one of the lamellar crystals of about 4 mm. length was chosen. The ends were clamped to silver electrodes as shown in Fig. 12, leaving about 2 mm. between. A movable

[1] Loc. cit.

slit of 0.1 mm. width was placed about 1 cm. in front of the crystal in the path of a parallel beam of light from an arc lamp. The slit was moved along in front of the selenium by a screw attachment. The conductivity was read when the illumination was on the contacts and at various positions between. The result showed the change of conductivity by constant illumination to be almost unvarying throughout the distance between the electrodes and to be slightly less when illuminated at the contacts. This result seems to lead unmistakably to the conclusion that in the crystal studied, light action is essentially a genuine action throughout the selenium crystal. The action was not a surface action, as evidenced by the fact that if the light impinged on the back side of the crystal the effect was unaltered.

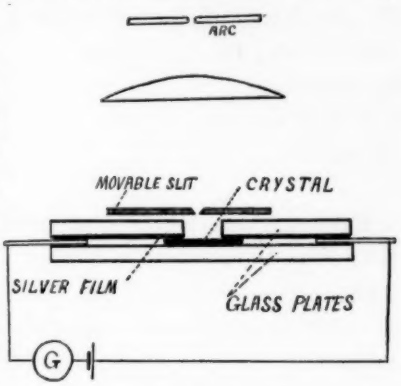

Fig. 12.

The Action of Light Spreads Inside of the Crystal.—With the arrangement shown in Fig. 12 the slit was adjusted midway between the two electrodes. Then the slit width was widened .05 mm. on each side at a step, the idea being that if each fraction of the light altered the part of the selenium in its path, that the change of conductivity would be in direct proportion to the width of the beam. This should follow from the fact that the parts of the selenium crystal were resistances in series. However the following is typical of the results obtained.

Slit Width Mm.	Convectivity.	Slit Width Mm.	Convectivity.
0	40	.4	101
.1	72	.5	109
.2	81	.6	118
.3	92	.7	126

This indicates that the first unit of light that falls on the selenium by diffused light in the selenium crystal or some sympathetic mechanism acts to either side of the path or throughout the crystal. Ever after that as the path is widened there is only the action of the selenium in the path or its equivalent. The first unit of light produces an excess of conductivity of 22 units, which can only be ascribed to a spreading action inside of the crystal or its equivalent. The above argument presupposes that not all the light absorbed in the selenium can go toward altering the conductivity, or further that the absolute alteration of the value of conductivity decreases with increased illumination.

The Variation of the Conductivity Change with the Light Intensity.—In the previous part of the paper, it was noted that the change in conductivity by a given illumination was proportional to the conductivity in the dark. This may be written in the form $dC/dI = k \cdot C$, providing that for very small illumination intensities the constant k should remain unchanged. By the integration of the above equation is obtained the value of the conductivity for any intensity, as $C = C_0 e^{kI}$, where C_0 is the conductivity in the dark.

I have not yet had the time to check this equation with more direct experiment.

ACTION OF TEMPERATURE.

Dieterich[1] has recently shown that the character of the wave-length sensibility curves can to a large degree be controlled by the heat treatment of the selenium during annealing, particularly by the temperature at which the selenium was heated. For example, he found that the annealing at about 200° produced a pronounced maximum in the red end of the spectrum, while if the annealing were about 150°, there was a relatively high maximum about .55 μ. Unfortunately I have not thus far been fortunate in producing large crystals by sublimation at both the above temperatures. Therefore we were not able to investigate individual crystals as to the character of the sensibility curves. However the next best step was taken. A selenium cell form (soapstone frame with parallel platinum wires for electrodes) was placed in a highly evacuated tube in which the selenium was to be sublimated. It was so situated lengthwise of the tube that at the end of higher temperature large crystals of the lamellar type formed in considerable abundance with smaller ones nearby the electrodes. About a centimeter further along the frame where the temperature was perhaps 200° was deposited an even finer crystalline structure of unknown texture. After remaining in the tube several days, almost the entire framework revealed in the presence of intense illumination a large number of glittering crystal faces, resembling in a striking manner the appearance of freshly fallen snow in bright sunlight. Except for the crystal faces in the selenium that were at just the right angle to the observer the background was relatively dark.

By the apparatus recently described by Brown and Sieg[2] we obtained the sensibility curve for the crystal masses at two places in this selenium bridge. For the crystal mass supposedly sublimated at the higher temperature, the upper curve in Fig. 13 was obtained, while for the selenium deposited at the lower temperature the lower curve in the figure was

[1] PHYS. REV., N.S., June, 1914.

[2] PHYS. REV., N.S., 2, p. 487.

obtained. Thus, contrary to Mr. Dieterich's result referred to, the red maximum appears in the crystals formed from the higher temperature. However it must be noted that this maximum is not in the same position as the maximum which Mr. Dieterich obtained at lower temperature. The cause of the shifting of the maximum will be searched for by the investigation of separate crystals.

GENERAL CONSIDERATIONS.

The present status of the theory of light sensitive selenium points towards a separtion of the complex elements along three lines, the light sensitiveness inherent in a given crystal structure, the varying action because of varying refraction, scattering and penetration, and the minor alterations due to the indirect action of the light on the contacts. In view of White's work it seems that contact resistances play a greater part in selenium blocks than in selenium crystals. Whether varying pressure will alter the character of the curves, will depend perhaps on the likeness or unlikeness of the light action for different parts of the spectrum. This point is under investigation at the present time. Also an attempt is being made to locate what characteristics of sensibility curves are seated essentially in individual crystal forms.

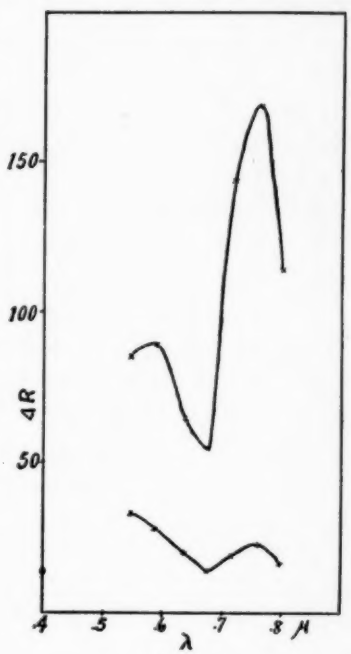

Fig. 13.

This much at least is settled: that the entire action of light in certain crystals at least is in the body of the selenium and not located either at the electrodes or at the surface layer.

Perhaps in the so-called selenium cells the varying temperature action as well as the varying pressure and light action are involved in connection with contact resistances. By placing high pressures, a few atmospheres, on a single crystal of selenium we should be able to eliminate this contact resistance and thereby arrive at the true temperature coefficient of a pure form of selenium and also the true light sensitiveness at various temperatures.

The action of light seems to be in centers inside the crystals and does not necessarily involve the existence of more than one complete crystalline

form. These centers inside the crystal resemble the so-called active centers in zinc sulphide as described by Rutherford.[1]

The specific conductivity of the crystals tested varied between 200 ohms and 10^7 ohms depending on the pressure applied. Judging from the conductivity, it would seem that the selenium in White's selenium blocks existed under an internal strain as a result of crystallization in a rigid form. If so we should expect different stress inside the blocks from what it was near the surface. Possibly some such basis might account for the increased photoelectric properties at the electrode contacts.

SUMMARY.

1. A large number of new crystals of metallic selenium have been formed, some of which are of very large size.

2. All of these forms except one are very transparent selectively to light, a large amount of light penetrating to a greater depth than 0.2 mm.

3. All the forms tested are conducting, showing a specific conductivity varying between 200 and 10^7.

4. All the crystal forms but one have been observed to be doubly refracting.

5. All the crystal forms increase in conductivity when illuminated.

6. The action of light is in the selenium itself and not at the contacts.

7. Mechanical pressure produces a genuine change in the selenium which may alter the conductivity more than a thousand times.

8. The absolute change of conductivity in one crystal by constant illumination was proportional to the conductivity in the dark, when that conductivity was altered by pressures between 1 and 180 atmospheres.

9. The temperature at which the crystals sublimate in mass has been shown to influence the character of the wave-length sensibility curves.

10. It has been shown that the production of individual crystals of metallic selenium of large size opens up a large field of investigation, which promises to be free from some of the possible complexities in selenium cells.

In conclusion I desire to thank Mr. M. H. Teeuwen for his assistance in designing and constructing apparatus, Mr. Scott Walker for assistance in taking observations, and Professor G. F. Kay, of the Geology Department, for the use of apparatus and also for his kind and generous help in the crystal studies.

THE PHYSICAL LABORATORY,
THE UNIVERSITY OF IOWA,
April 11, 1914.

[1] Proc. Roy. Soc., *A*, 83, p. 561, 1910.

ON PONDEROMOTIVE FORCE UPON A DIELECTRIC WHICH CARRIES A DISPLACEMENT CURRENT IN A MAGNETIC FIELD.

By Robert H. Goddard.

Introductory.

According to Lorentz's theory of a material dielectric as being composed of elementary charges, such a body should, when carrying an electric displacment current in a magnetic field, experience a ponderomotive force similar to that which acts upon a conductor when carrying a conduction current in a magnetic field. Also, a ponderomotive force should be present in the analogous case of a body, of permeability greater than unity, when carrying a magnetic displacement current in an electrostatic field. The present paper deals with these two researches, neither of which has hitherto been successfully performed.

An attempt made by Whitehead[1] to demonstrate the presence of the first of these forces has given a negative result. Lorentz,[2] R. Gans,[3] and Koláček,[4] have shown this fact to be in agreement with electrical theory, without however suggesting a way by which a positive result could be attained.

Theory of the Ponderomotive Action for Electric Displacement in a Magnetic Field.

(*a*) *General Theory.*—The following theory shows that the Whitehead experiment is but a particular case of a more general research, a statement of which will now be made. Let there be given a vertical magnetic field, H, produced by two co-axial, circular, coils, in which an alternating current is flowing. Suppose a strip of homogeneous dielectric, D, Fig. 1, to be suspended from an arm, a, which is pivoted at an axis O; the strip being in the said magnetic field, between the upper and lower coils.

Suppose, further, that condenser plates, K_1 and K_2, are placed upon each side of the strip, at right angles to the arm, a; there being impressed upon these plates an alternating E.M.F., of the same period as the current

[1] J. B. Whitehead, Jr., American Journal of Science, 4 ser., Vol. 14, pp. 109–128, 1902.

[2] H. A. Lorentz, Konink. Akad. van Wetten. te Amsterdam, Vol. 5, pp. 622–628, 1903.

[3] R. Gans, Phys. Zeit., Vol. 5, pp. 162–4, 1904.

[4] F. Koláček, Phys. Zeit., Vol. 5, pp. 45–47, 1904.

in the coils, but differing in phase by a quarter period. Both the displacement current in the dielectric, and the magnetic field, will thus be in the same phase. For experimental reasons the condenser plates K_1 and K_2, and the surfaces of the strip are taken as cylindrical surfaces concentric about O; the radii being large compared with the thickness of the dielectric strip. Let distances, measured outward along the radius, be r; and counter-clockwise along the arc, be s. Consider an element, $drds$, of the dielectric strip.

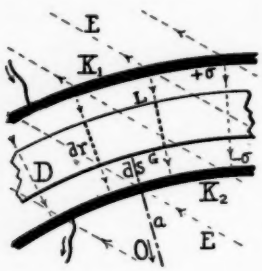

Fig. 1.

It is required to find the conditions, if any exist, for the presence of a moment of force, about O, due to the electric displacement in this element, $drds$, in the presence of the magnetic field—analogous to the force upon a conductor which carries a conduction current in a magnetic field. Before proceeding to a discussion of the moments which must act upon the element,[1] a few preliminary remarks are necessary. Let the surface density be σ at K_1, and $-\sigma$ at K_2. The charges upon K_1 and K_2, at the ends of the imaginary stream-tube which passes through the element, will then be σds and $-\sigma ds$, respectively; if we consider, for convenience, that the vertical width of the strip is one centimeter. We shall then have a current in the lead-wires of the condenser, due to the continuation of this elementary current tube,[2] of magnitude

$$i_w = \frac{d\sigma}{dt} ds. \tag{1}$$

Let the dielectric in question be of constant, k, and, for simplicity, be surrounded by a medium of constant unity; and let G be the element of surface on the inner, and L the element on the outer, boundary. Then, inasmuch as the dielectric is homogeneous,[3] the circumstances will remain identically the same if the dielectric is removed, and charges are given the boundaries G and L of respective densities

$$\sigma_G' = \frac{k-1}{k}\sigma, \tag{2}$$

and

$$\sigma_L' = -\frac{k-1}{k}\sigma. \tag{2'}$$

[1] Lorentz, *loc. cit.*, derives expressions for the *reaction* of the moments upon the condenser plates. The expressions, however, hold only for the case in which the dielectric completely fills the space beteen these plates.

[2] After Maxwell's fundamental conception of a *closed circuit* as being a conduction, plus a displacement, current.

[3] Were the dielectric heterogeneous, it would be necessary also to consider apparent bodily charges.

These charges are the so-called apparent charges, to which the presence of the dielectric is equivalent.

Let us now define a *virtual* or *apparent current*,[1] in the space occupied by the dielectric, as the *time-rate of change of apparent density of charge at the boundary*. We shall then have for the apparent current across the element

$$i_v = \frac{d\sigma'}{dt} ds = \frac{k-1}{k} \frac{d\sigma}{dt} ds, \tag{3}$$

this current being in the same direction as that in the lead-wires of the condenser.

Moment I.—The moment of force upon the dielectric element, due to this apparent current, together with the vertical magnetic field H will be

$$dM_1 = \frac{a(k-1)}{ck} \frac{d\sigma}{dt} Hdrds, \tag{4}$$

where c is the ratio of the electrostatic to the electro-magnetic units.

Moment II.—Another couple is to be considered, owing to the combined presence of the apparent charges produced by the condenser K_1K_2, and the electrostatic field which must accompany the varying vertical magnetic field, H. This electrostatic field is represented in direction and intensity by the vector E, Fig. 1. It is directed in circles about the axis of symmetry of the coils which produce the magnetic field. We may take the point (rs) as coinciding with the lower right-hand corner of the dielectric element.

The moment due to the apparent charge at G is the product of the apparent charge at G by the moment of the s-component of E (*i. e.*, aE_s).

By equation (2) this is

$$\frac{k-1}{k} a\sigma E_s ds.$$

The corresponding moment, due to the apparent charge at L is, by (2′),

$$-\frac{k-1}{k} a\sigma \left(E_s + \frac{dE_s}{dr}\right) ds,$$

making the total moment

$$dM_{II} = -\frac{k-1}{k} a\sigma \frac{dE_s}{dr} drds. \tag{5}$$

[1] This apparent current corresponds to the "polarization current," and should not be confused with the conception of "apparent current" of which use is made in the theory of media, heterogeneous with respect to permeability. It will be noticed that the present treatment makes use of the fact that the presence of the dielectric is equivalent electrostatically to apparent charges on the boundaries; and magnetically, to the polarization current.

Moment III.—It is, of course, necessary to consider apparent charges as being produced by the field E. The apparent densities, let us say σ'', at G and L will be, respectively,

$$\sigma_G'' = -\frac{k-1}{4\pi k} E_r \tag{6}$$

and

$$\sigma_L'' = \frac{k-1}{4\pi k}\left(E_r + \frac{dE_r}{dr}dr\right). \tag{6'}$$

An apparent current, as above defined, will then take place in the space occupied by the dielectric, in the r-direction; and this, together with H, will give a moment, to the first order, of

$$dM_{III} = \frac{(k-1)a}{4\pi ck}\frac{dE_r}{dt} Hdrds. \tag{7}$$

It is important to notice that the mean value of dM_{III} is other than zero, for the reason that H and dE/dt differ in phase by a half period.

Moment IV.—The apparent charges at G and L produced by the field E must be considered as being acted upon by the field E itself.

The moment of the force E upon the apparent charge at G is

$$-\frac{(k-1)a}{4\pi k} E_r E_s ds,$$

and that upon the apparent charge at L is

$$\frac{(k-1)a}{4\pi k}\left(E_r + \frac{dE_r}{dr}dr\right)\left(E_s + \frac{dE_s}{dr}dr\right)ds,$$

so that the total moment is

$$dM_{IV} = \frac{(k-1)a}{4\pi k}\left(E_s\frac{dE_r}{dr} + E_r\frac{dE_s}{dr}\right)drds. \tag{8}$$

It remains to apply this theory to particular cases, in which the piece of dielectric is of a size sufficiently large to permit of experimentation.

(*b*) *Application to Particular Cases. Case* 1.—In the Whitehead experiment,[1] a small block of dielectric was suspended between two condenser plates, the arrangement being analogous to the element *drds*, with the field E wholly in the s-direction.

Although the case is different from that in which the element is a part of a strip, it is nevertheless possible to determine, in a general way, what moments will be effective. The moment dM_{II} is equal to the line-in-

[1] Whitehead, loc. cit. Three other forms of apparatus were tried by Whitehead, in which the arrangements were such that the field, E, was directed in circles concentric with the center of the dielectric block. In each of these forms, however, there was present a comparatively strong electrostatic controlling force.

tegral, taken in the positive direction of rotation, of the force E_s, about the contour of the element (*i. e.*, *curl* E) multiplied by the apparent density, $-[(k-1)/k]\sigma$. This is true, owing to the fact that at the ends, dr, of the element, both the apparent charges, and E resolved tangentially, are zero. But this contour integral of the tangental component of E is, by Maxwell's equation,

$$\text{curl } E = -\frac{1}{c}\frac{\partial H}{\partial t}\,drds, \tag{9}$$

for it will be noticed that $Hdrds$ is the flux of the magnetic field within the contour. Moment dM_{II} thus becomes

$$dM_{II} = \frac{(k-1)a}{ck}\sigma\frac{dH}{dt}\,drds. \tag{10}$$

The sum of moments I and II is, then, by (4) and (10)

$$dM_I + dM_{II} = \frac{(k-1)a}{ck}\frac{d}{dt}(\sigma H)drds. \tag{11}$$

Since this is the time-rate of change of a periodic quantity, the mean value will be zero. Furthermore, since $E_r = 0$, moments dM_{III} and dM_{IV} will be zero,[1] whence the total moment will vanish—which accounts for the negative result of the Whitehead experiment.

Case 2.—If a strip of dielectric is employed, instead of a small block, the theory here presented holds strictly true for any given element; and it is at once evident that, if the element is so placed in the field, E, that E_s is zero, then moment dM_{II} will vanish. If, at the same time,moments dM_{III} and dM_{IV} can be eliminated, there will remain only moment dM_I—which is the moment desired.

In considering the resultant moment for the entire strip, it is evidently desirable that the integral of dM_I with respect to s should be a maximum, and the sum of the integrals of dM_{II}, dM_{III} and dM_{IV}, should be a minimum. At the same time, it should be borne in mind that the strip must have as boundaries, surfaces cylindrical about the axis O, in order to permit of a rotation about this axis, during the experiment.

These conditions are satisfied, as nearly as possible, by the strip of dielectric shown in Fig. 2. The vertical width of the strip is moderate, which permits two coils being placed close to it, one above and the other beneath. The diameters of these coils do not exceed the length of the condenser K_1K_2, so that the magnetic force is inconsiderable beyond

[1] In the Whitehead experiment, the ends, dr, of the dielectric, must be considered as having had apparent charges due to the force E_s; but owing to symmetry, E was everywhere the same within the immediate neighborhood of the dielectric, for any particular value of r. The resultant moment due to these apparent charges, was, therefore, zero.

the dotted circles of electric force, E. The strip is sufficiently thin for the thickness, l, to correspond to dr; it thus being unnecessary to express integration with respect to r. With this arrangement, the various moments are as follows.

Moment I.—Owing to the magnetic field being everywhere vertical, and the displacement current between the condenser plates being everywhere radial with respect to O, each element experiences the total

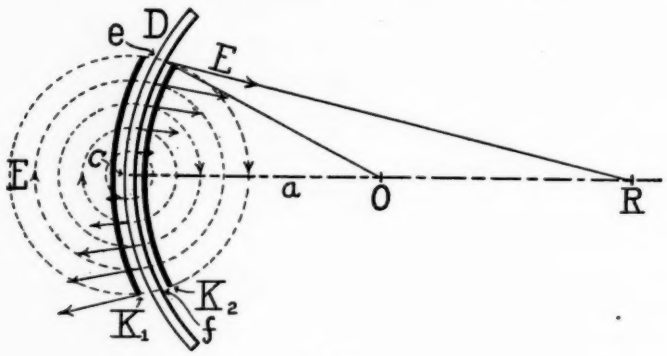

Fig. 2.

moment dM_I, so that the moment for the entire strip (or, what is equivalent, the part of the strip within the condenser) is

$$\int_e^f \frac{dM_I}{ds}\,ds = \frac{a(k-1)}{ck}\,\frac{d\sigma}{dt}\,Hsdr,$$

or

$$= \frac{a(k-1)}{ck}\,iHl, \qquad (12)$$

where i is the total current through the strip.

Moment II.—The second moment would vanish completely if E were everywhere normal to the boundaries of the strip, and also if there were no apprent charges at the ends. The force, E, is not normal, but (for an infinitely thin strip) passes through the point R, distant $2a$ from c, instead of through the point O.

There are, however, *no apprent charges at the ends* of the strip, owing to the fact that the ends project into a region in which both the fields E, and that due to K_1K_2, are negligible. These projecting parts of the strip are of importance also in another respect; namely, in permitting the strip to swing through a small angle, about the axis O, without experiencing an electrostatic controlling force tending to keep it symmetrical with respect to the condenser K_1K_2.

An expression for the integral of dM_{II} may be derived in the following

way. The strip may be considered as ending at the points e and f, inasmuch as the fields of force do not extend beyond these points. Let us denote by ϵ the ratio of the line integral of E resolved tangentially along the *sides* of the strip, to the line integral of E resolved tangentially along the *sides and ends* of the strip. Then, making use of (9), as before, and integrating, we have

$$\int_e^f \frac{dM_{II}}{ds} ds = \frac{\epsilon(k-1)a}{ck} \sigma \frac{dH}{dt} ls, \tag{13}$$

which, when combined with (12) gives for the sum of the first and second moments,

$$(1-\epsilon)\frac{(k-1)a}{ck}\frac{d\sigma}{dt}Hls+\epsilon\frac{a(k-1)}{ck}\frac{d(\sigma H)}{dt}ls=(1-\epsilon)\frac{(k-1)a}{ck}\frac{d\sigma}{dt}Hls. \tag{14}$$

A method of measuring $(1-\epsilon)$ will be given below under "Details of Method and Apparatus."

Moment III.—It is evident that the radial components of E are equal in magnitude but opposite in direction, at points on the boundary, equal distances on each side of the center, c, The integral of the third moment thus vanishes, by symmetry.

Moment IV.—From what has just been said concerning Moment III., it is evident that the apparent charges produced by the radial components of E are equal in magnitude, but opposite in sign, at points equally distant on each side of c. Further, it may be seen from the figure that the tangential component is in the same direction at all points of the strip, and is of equal magnitude at equal distances from c. Hence the integral of the fourth moment also vanishes.

The sum of all four moments for the entire strip thus reduces to the expression (14).

THEORY OF THE PONDEROMOTIVE ACTION FOR MAGNETIC DISPLACEMENT IN AN ELECTROSTATIC FIELD.

The theory of ponderomotive action herein developed will apply equally well to a body, of permeability greater than unity, which carries a magnetic displacement current in an electrostatic field. In this case, in place of the two coils, previously described, there must be substituted a condenser consisting of two plane, circular, plates, placed co-axially, one above the other; and in place of the condenser K_1K_2, there must be substituted two coils of such forms as will give a field of force similar in shape to that of this condenser. The strip corresponding to D might be composed of fine iron filings imbedded in paraffine.

The case of electric displacement in a magnetic field, is, however,

more amenable to accurate experimentation; and a research will now be described in which the theory here presented is applied to a system such as that shown in Fig. 2.

GENERAL METHOD.

Two difficulties which were encountered in the Whitehead experiment, namely, electrostatic disturbing forces in an apparatus not perfectly symmetrical, and the impossibility of using a comparatively large mass of dielectric, were both overcome in the present experiment by the use of currents of very high frequency.[1] It was thus possible to secure a current of several tenths of an ampere through a condenser of surprisingly small capacity. A strip of dielectric, supported on the arm of a torsion balance, was permitted to swing freely between the plates of this condenser.

A measured value of the force was obtained from the constant of torsion of the fiber, together with the deflection of the mean position during torsional vibrations, produced by a reversal of the force.

A calculated value of the force was obtained from the following measurements: the thickness of the dielectric strip; the mean magnetic field, per unit current, within the space occupied by the dielectric; the dielectric constant at the particular frequency employed; the factor $(1 - \epsilon)$; and the proportion of total current that passed through the strip.

DETAILS OF METHOD AND APPARATUS.

(*a*) *Coils and Condenser.*—The magnetic field which acted upon the

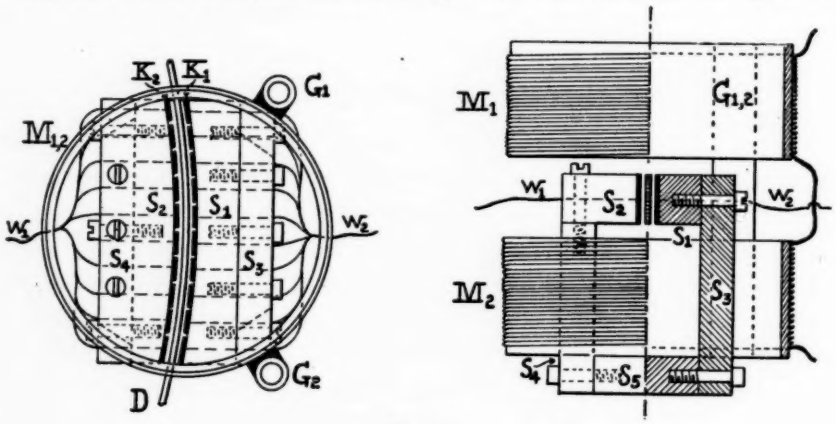

Fig. 3.

Coil and Condenser. ½ actual size.

dielectric was produced by two coils shown as M_1, M_2, in the plan and elevation of the coil-condenser apparatus, Fig. 3. These coils were

[1] The frequency in Whitehead's experiment was 133 cycles per second.

2.2 cm. apart, and each consisted of a single layer of 28 turns of cotton-covered copper wire, 1 mm. in diameter, wound on a section of glass tube 7.45 cm. in diamater. They were soaked in melted paraffine, after being wound.

The condenser K_1K_2, in which the dielectric swung, was composed of two rows of separate brass segments, in order to eliminate eddy-currents otherwise present from the alternating magnetic fields of M_1 and M_2; each segment being connected by a wire with the leads, w_1 and w_2. The rows K_1 and K_2, were of 1.3 cm. vertical width and were separated by an air-gap of 0.4 cm. The surface of each row was cut accurately to the arc of a circle, with center distant 20 cm. from the middle of the air-gap.

The coils and condenser were held in position by a support consisting of five pieces of hard rubber, $S_1 \cdot \cdot \cdot S_5$, fastened with hard rubber screws. The coil M_1 was fastened with sealing-wax to two glass rods G_1 and G_2, which were, in turn, fastened to M_2.

(*b*) *The Suspended System.*—The torsion arm, *B*, Fig. 4, consisted of a glass tube 0.25 cm. in diameter and 40 cm. long weighing 0.09 grains per linear cm., with a lead weight, *W*, at the one end, and two branches which supported the dielectric, *D*, at the other end.

The dielectric, used in the experiment to be described below was a strip of hard rubber 15 cm. long, 0.13 cm. thick, and the same vertical width as the condenser K_1K_2; namely, 1.3 cm. It was given the proper curvature by being heated, and placed while hot upon a cylindrical surface of brass, of 20.065 cm. radius. The strip projected 4 cm. beyond the ends of the condenser K_1K_2.

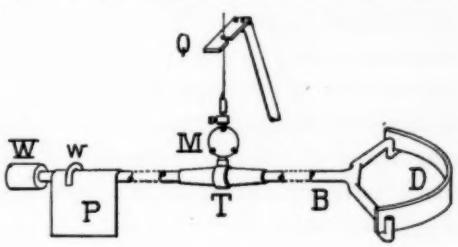

Fig. 4.
The Suspended System.

A mirror, *M*, reflected the image of a vertical wire, illuminated from behind by a Nernst filament, to a scale 160 cm. distant. It was found later, however, that greater accuracy could be attained by observing an illuminated transparent scale, distant 3 meters from *M*, through a telescope magnifying 70 diameters.

The suspended system was adjusted for level while free from the coil-condenser apparatus. The arm *B* was made level by means of the 2 gram rider, *w*; while, at the same time, the distance between the axis of suspension and the middle of the strip was made 20 cm., by calipers—this latter adjustment being made by moving *B* lengthwise in the brass holder *T*.

The upper edge of the strip was made level by turning the glass tube in the holder T until this edge remained, during torsional vibrations, at a constant small distance from a fixed needle-point, held just above the strip.

The proper height of the strip was easily secured by means of the wire, supporting the fiber, which passed through a torsion-head of the ordinary type. The height of this wire was varied until the top edge of the strip D was level with the top of the condenser K_1K_2.

The strip was made to swing exactly in the middle of the air-gap, in the following way; a template was made, with one edge cut to the arc of a circle of 20.2 cm. radius, and with a small hole through the point corresponding to the center of this circle. Next, the piece S_2, Fig. 3, was removed, and the curved edge of the template was brought in contact with the row of segments K_1. A fine wire with a small weight attached, supported by the torsion head, was passed through the hole in the template; and the torsion head was then moved until the wire, handing freely, passed through the center of this hole.

In order subsequently to place the suspension fiber in exactly the position occupied by this wire, a guide Q was made, as follows: a small hole was drilled in a flat plate and a slot was cut from the edge of the plate to the hole, as shown in the figure. A rod was fastened to this plate, and was held in such a position that the wire, above mentioned, passed through the center of the hole. While in this position, the lower end of the rod was fastened, with sealing-wax, to a glass plate, 60 cm. square, which supported the coil-condenser apparatus.

(*c*) *The Suspension.*—Quartz fibers were found more satisfactory than phosphor-bronze strip. The fibers used were 0.05 mm. in diameter, 30 cm. in length, and were cut from fibers 4 meters long, produced by the bow-and-arrow method.

The torsion-head was held in an iron clamp, supported by an apparatus-stand of the ordinary pattern. This stand rested on the same wooden table as the other apparatus. The means for producing the oscillations was on a separate table, as far from iron, and other conductors, as possible.

(*d*) *Elimination of Air-currents.*—The suspended system and coil-condenser apparatus were enclosed in a box of cardboard, 52 cm. square and 21.5 cm. high. This box was double-walled, 3 cm. thick, the interspace being packed with cotton; and the top was made in sections, so as to permit removal without disturbing the suspension. Within this box was a small single-walled box, which surrounded closely, but without touching, the coil and condenser, and the suspended system. Both boxes, as well as the coil-condenser apparatus, were fastened to the above-

mentioned glass plate, being so placed that the fiber was over the middle of this plate. Small windows of plate glass permitted observation of the mirror, M; and the fiber suspension was enclosed in a glass tube, fastened to the torsion-head.

After the apparatus had been adjusted, it was covered completely with several layers of cotton; but even with this protection, it was found necessary to keep the temperature of the room constant by means of a thermostat. When this was done the zero was very steady.

Owing to the fact that the coils produced heat close to the dielectric strip, it was necessary to enclose the coils and lead-wires completely with heavy cardboard—the condenser alone remaining exposed. Two openings, into the space thus enclosed, served to remove the heated air, by convection.

In order to set the suspended system into vibration, when completely enclosed, a strip of paper, P, Fig. 4, was fastened to the arm B. A minute jet of air could be directed upon this strip in the following way. A glass tube, with the end drawn to a fine opening, was fastened in the walls of the boxes, with the small end directed toward P. A rubber tube, closed at one end, was attached to the end of the glass tube that projected from the boxes. By gently pressing this tube, a torsional vibration of any desired small magnitude could be produced.

It was found that when the apparatus was set up in cold weather, an electrostatic charge was acquired which manifested itself in a strong controlling force. This difficulty was overcome by placing, for a few hours, a half milligram of radium bromide in a glass tube, within the double-walled box.

(*e*) *Source of High-frequency Current.*—The alternating current was produced by a Chaffee arc, or gap. This consists essentially of an aluminum cathode and a copper anode, surrounded by an atmosphere of moist hydrogen. The air-cooled gap described by Chaffee[1] was used with modifications of certain mechanical features.

Details of the gap are shown in Fig. 5; the right half of the figure being in section. At e is shown the aluminium electrode which fitted tightly in a brass rod, or terminal, t. This terminal was free to slide in a brass tube b; which latter was provided with radiating vanes, as shown. Opposite the aluminium electrode was a copper electrode, held in a similar manner. The terminals were provided with hard-rubber handles r, for adjusting; and were fastened, when adjusted, by long screws with hard-rubber heads, s.

The brass tubes, b, were threaded, and engaged in the side-plates, c;

[1] E. L. Chaffee, Proceedings, American Acad. of Arts and Sci., Vol. 47, pp. 286–312, 1911.

which latter were fastened to a hard-rubber base. Holes were drilled in one side-plate to provide an inlet, *i*, and an outlet, *a*, for the hydrogen produced electrolytically by a current of 3 amperes. The side-plates were separated by a hard-rubber ring, *h*.

This form of the gap was of advantage in that the ring of hard-rubber was free from inlet and outlet holes, and could therefore easily be re-

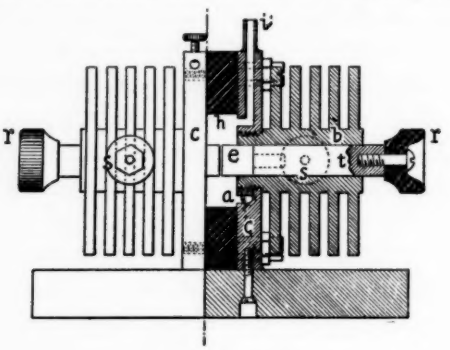

Fig. 5.

Air-cooled Chaffee Discharge Gap. ⅓ actual size.

placed; and, further, the terminals were of a shape convenient to be held in a chuck whenever it became necessary to expose fresh, plane, surfaces at the ends of the electrodes.

(*f*) *Arrangment of Connections.* The connections are shown in the diagram, Fig. 6. The source of direct current for the gap, *G*, consisted of two 250-volt generators, in series, one generator being provided with

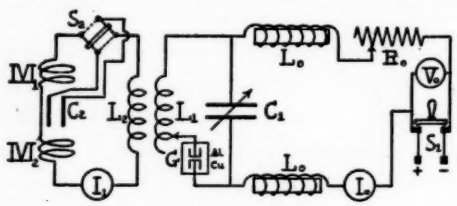

Fig. 6.

Diagram of Connections.

a field-rheostat to regulate the voltage. These generators delivered 530 volts at the main switch S_1, as read by the volt meter V_0. Owing to electrostatic charges from the generators, produced by belt-friction, it was found necessary to connect one point of S_1 to earth through a long glass tube filled with dilute copper sulphate solution.

A direct current ammeter I_0 measured the current in the primary circuit. This circuit also contained two inductances or choke-coils, L_0

(ordinary alternating current arc-light inductances), together with a variable resistance and a bank of lamps in series, R_0. The current was reduced to about 0.2 ampere, by means of these resistances.

The primary condenser, C_1, consisted of two circular brass plates, in air, 30 cm. in diameter, and 0.2 to 0.5 cm. apart. Adjustment of this latter distance was made by a micrometer screw. The primary inductance, L_1, was a single layer of 1 mm. diameter copper wire, cotton covered, wound on a glass tube, 1.4 cm. diameter and 3.5 cm. long. The secondary, L_2, was wound directly over these primary turns, after the latter had been dipped in melted paraffine. Close coupling, as just described, was found necessary, as otherwise the secondary current was very irregular. Adjustments of R_0, C_1 and the potential V_0, were made until the secondary current was a maximum.

A commutator, S_2, served to reverse the secondary current, usually 0.3 or 0.4 ampere, through the condenser C_2 (corresponding to K_1K_2 of Fig. 3). This commutator was of small capacity, and consisted of four small mercury cups at the top of four glass rods, 10 cm. high. Even with this arrangement there was a noticeable change in frequency, on reversing, due to a change in the area enclosed by the secondary circuit, and a consequent change of inductance.

The hot-wire ammeter, I_1, was an R. W. Paul high-frequency galvanometer, having sufficient resistance in series with the moving coil to give a complete scale deflection with 0.5 ampere. This instrument was purposely placed between L_2 and the coils M_1M_2, in order to be as near the node of potential as possible. This was done to reduce to a minimum the capacity of I_1 to earth. It should be remarked, in passing, that the secondary current produced no sensible heating of the dielectric strip.

(*g*) *Measurement of the Magnetic Field.*—It was found possible to measure directly, with a sufficient degree of accuracy, the mean magnetic field, in the space occupied by the dielectric, per unit current in the coils, M_1, M_2. A glass strip, the same width and thickness of the dielectric, and of a length *ec*, Fig. 2, was given the same curvature as the dielectric. Upon this strip were wound eighty turns of fine, enamelled, copper wire. The resulting search-coil was fastened with wax, in the gap of the condenser K_1K_2, with one edge at *e* and the other at *c*; thus filling the space occupied by half the dielectric. The flux of the magnetic field through this coil was, by symmetry, half the flux through the entire dielectric strip.

This flux was measured with a Grassot fluxmeter, by using as much current as the coils M_1M_2 could safely stand; namely, 7 amperes. The coils M_1M_2 were, of course, first disconnected from the secondary circuit,

Fig. 5, and the dielectric was removed from the condenser K_1K_2. The arrangement of connections was such that the deflection of the fluxmeter resulted from a *reversal* of the current in the coils M_1M_2, rather than from applying and removing this current; thus affording a deflection of twice the magnitude obtainable in the ordinary way.[1]

The fluxmeter was calibrated by using a circular search-coil, of 15 turns, 3.80 cm. in diameter, placed in the middle of a solenoid. This solenoid carried a current of 4 amperes, and consisted of a single layer of cotton-covered copper wire wound upon a glass tube. The constants were: length, 49.2 cm.; mean diameter of windings, 5.6; and number of turns per cm., 1.063.

It was necessary to determine the area-turns, or effective area, of the curved search-coil already described. This was done by comparing the two search-coils in the field of an electromagnet. The electromagnet employed for this purpose was one used by Professor O. W. Richardson in determining the constants of thermions, which he kindly placed at the writer's disposal; the field of which had been found to be very uniform over a considerable area. The effective area of the search-coil found in this way was 52.25 cm. It should be remarked that all the measurements recorded were means of a number of determinations, in close agreement, taken at various parts of the fluxmeter scale.

The value of H thus obtained, per unit current in M_1M_2 was 5.23, to within 1 per cent. A test of the flux, in the space occupied by the part of the dielectric strip *outside* the condenser K_1K_2, was made with the curved search-coil. It was found that this flux was but 2 per cent. of that within the gap of the condenser.

(*h*) *Measurement of* $(1 - \epsilon)$.—A measurement of $(1 - \epsilon)$ was made possible, for the apparatus under discussion, by the fact that the dimensions of the coils M_1M_2 were small compared with the wave-length (100 meters). For this reason, all the parts of the magnetic field with which we are concerned were, at any instant, in the same phase. Thus from the theorem

$$\text{curl } E = -\frac{1}{c}\frac{\partial H}{\partial t}\,drds, \tag{9}$$

we have

$$\text{curl } E \propto Hdrds. \tag{15}$$

Now, since the apparent charges exist only between the condenser

[1] A cylindrical commutator was devised which automatically performed this operation in the proper order. The deflection could thus be magnified n times, by giving the commutator n turns. It was found, however, that there was too rapid a drift toward the zero to make this method practicable, owing to the short free period of the fluxmeter (one minute). With a free period of 15 or 20 minutes, satisfactory results could have been obtained.

plates K_1K_2, on the inner and outer boundaries of the dielectric, we may assume, as already stated, that the dielectric strip does not extend beyond the points e and f, Fig. 2; and also that the ends are portions of the circumference of a circle passing through e and f, with center at c.

The ratio of the part of *curl E* eliminated to the total *curl E* for the strip,—in other words, $(1 - \epsilon)$—will then be by (15)

$$(1 - \epsilon) = \left(\frac{a}{b} f_c\right) \frac{1}{f_d}, \tag{16}$$

where a = the sum of the arcs of the circle, cut by the dielectric strip at e and f (0.26 cm.),

b = the circumference of this circle (21.67 cm.),

f_c = the flux of H within this circumference, for a given current in the coils M_1M_2, and

f_d = the flux of H within the boundaries of the dielectric strip, for the same current in M_1M_2.

An experiment was thus made necessary to determine H as a function of the distance from the axis of symmetry, c, of the coils M_1M_2. This was done by measuring the flux, as described under (g), through five circular search-coils, for the same current in M_1M_2; each coil being placed with the center coinciding with c. The measurements made with these coils, reference to one of which has already been made, are given in the following table.

Coil No.	Diameter.	No. of Turns.	Fluxmeter Deflection.	Nos. $\propto \frac{\text{Flux of H}}{\text{Turns} \times \text{Circum.}}$
1	1.275	147	33.2	0.566
2	2.540	33	37.6	1.425
3	3.80	15	37.4	2.08
4	5.08	8	36.5	2.845
5	6.32	5	35.5	3.56

From the numbers in the last column it will be noticed that for a single turn, the flux of H, for each search-coil, divided by the corresponding circumference, is a linear function of the distance from the axis of symmetry, c; as shown clearly by Fig. 7. Hence E, by (15), is also a linfunction of this distance; and H is therefore practically the same at all points in the region considered.

It thus becomes possible to substitute for the flux f_c through the circle in question, and the flux f_d within the boundaries of the dielectric, the respective areas enclosed by these figures; namely, 37.37 cm.^{2} and 0.963 cm.^{2} The ratio (16) is then found to be 0.4659. This is the ratio of

the part of *curl E* at the *ends* of the strip, where there are no charges, to the *total curl*; it is thus the fraction of total *curl* which does not give a perfect differential, with respect to the time, when (13) is added to (12); and is, therefore, the quantity $(1-\epsilon)$.

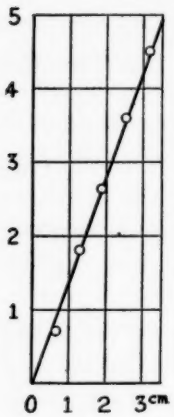

Fig. 7.

Ordinates; numbers proportional to *E*. Abscissæ; radii of search coils

(*i*) *Measurement of Dielectric Constant.*—The apparatus used in the measurement of dielectric constant was the condenser shown in Fig. 8, together with a wave-meter of the Dönitz type. The supports of the condenser plates, P_1P_2, were hard rubber throughout. The plates were 4 cm. in diameter and 1 mm. thick. P_1 was fixed whereas P_2 could be moved by a micrometer, *N*. The hard rubber rod, *r*, which supported the movable plate, traveled in brass bushings, *b*, and was provided at the end with a steel ball, kept pressed against the micrometer under the influence of the spring, *s*. This rod was prevented from turning by a pin which moved in a groove in one of the hard rubber supports. The dielectric to be examined, in the form of a plate or disk, *D*, was fastened to a hard rubber slider, *J*, free to slide lengthwise on the base of the instrument.

The wave-meter consisted of a variable air condenser of semi-circular plates, by Cossor, of 0.003 microfarad maximum capacity, together with an inductance. The latter was a single loop of copper rod, 81 cm. by 23.5 cm., attached to the terminals of this condenser. Across these

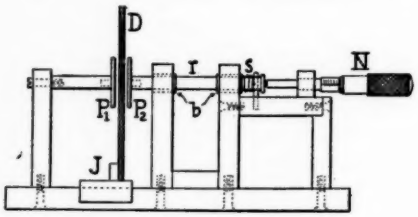

Fig. 8.

Condenser for Measurement of Dielectric Constant. (⅓ actual size).

terminals were connected, in series, a 1,000 ohm telephone receiver and an aluminium-tellurium rectifier, after Austin.[1]

This type of rectifier was found much superior to a silicon-steel contact, at the frequencies used; and a loud hissing sound could be heard in the telephone while the pointer was passing over about 40′ of arc. Such sharp tuning as this was only possible when the wave-meter was at least 4 meters from the oscillating system.

[1] L. W. Austin, Bulletin, Bureau of Standards, Vol. 5, 1, pp. 133–147.

The procedure consisted in substituting, in the secondary circuit shown in Fig. 6, the condenser just described and a suitable small inductance, in place of C_2 and M_1M_2. The wave-meter was kept set for resonance at the frequency of the secondary, with C_2, M_1M_2, and the dielectric strip in position. The micrometer was adjusted until this frequency was attained by the new secondary circuit. The dielectric was then removed from between P_1 and P_2, and the micrometer adjusted until the frequency was exactly the same as before. Since no inductance was changed in the process, and further, since the frequency was the same in both cases, the capacities must also have been the same. If we denote by k, the dielectric constant; d_1, the thickness of the dielectric; and d_2, the difference in the two micrometer readings, we have

$$k = \frac{d_1}{d_1 - d_2}. \tag{17}$$

This method assumes that the field within the dielectric is everywhere parallel to the axis of the condenser plates; so that the removal of the dielectric and adjustment of the plates until the frequency becomes again the same will amount merely to the substitution of an equivalent thickness of air. It will be noticed that, with the condenser P_1P_2, the current density was of the same order as that in the experiment with the movable dielectric strip.[1]

(*j*) *Proportion of Total Current in the Dielectric Strip.*—In order to obtain an idea of the magnitude of the distributed capacity together with certain other leakages, experiments were first performed to find the proportion of the total current, as measured by I_1, that passed through the dielectric strip, when the coils M_1M_2 were removed from the immediate neighborhood of K_1K_2—the electrical connections remaining the same. A curved glass strip, of the same shape and size as the hard rubber strip used in the actual measurements, was placed in the gap and then withdrawn—the change in capacity of the wave-meter necessary to produce resonance being noted.

The corresponding readings, in order, were 45° and 37° 30′, showing that the increase in capacity due to the glass strip was 20 per cent. of the original capacity. The change in capacity, calculated from the thickness of the strip and the dielectric constant ($K = 2.55$ by the method previously described) was 26.4 per cent. In this calculation the dielectric was considered as being part of an infinite plate, between

[1] In measurements of dielectric constants, in other researches, in which small current-densities are employed, it would be advantageous to use several condensers in series, each similar to that shown in Fig. 8, with plates of large area, close together. In this way the end corrections would be small, while at the same time the frequency could be very high.

condenser plates of infinite extent. From this it follows that 75.8 per cent. of the total current, as measured by I_1, passed through the dielectric. In other words, the loss of current due to the part of the field of K_1K_2, lying outside the strip, together with the distributed capacity of M_1M_2, and the capacity to earth of the hot-wire ammeter, I_1, was but 24.2 per cent.

The proportion of the total current through I_1 which passed through the strip while the measurements of ponderomotive force were being made was found by repeating the preceding experiment after M_1 and M_2 had been replaced in their normal positions, as shown in Fig. 3. In this case, the introduction and removal of the glass strip caused a change in wave-meter capacity of from 84° to 78°. Thus the total capacity was now greater than before, whereas the proportion of total current passing through the dielectric was less. The proportionate increase in capacity due to introduction of the glass strip was 7.6 per cent. so that but 28.8 per cent. of the total current passed through this strip.

It was hoped that each of the factors which enter into the expression for the ponderomotive force could be measured to within 1 per cent. The corrective factor under consideration is important, but unfortunately it involves the most difficult of all the measurements—depending as it does upon a *difference* of reading of but a few degrees—each reading not being made closer than 20′. Thus the error of this measurement exceeded that of all the others combined, except under the very best conditions of tuning of the wave-meter.

It was assumed that the fractional change in capacity, on introducing and removing the strip, was due solely to an influence upon those electrostatic lines which passed through the strip. This, however, was not the case, as there was necessarily present a bending, toward the upper and lower edges of the dielectric strip, of the electrostatic lines from the condenser K_1K_2 which passed immediately above and below. It is thus seen that a part of the increase of 7.6 per cent. in capacity was due to displacement currents lying outside the dielectric strip, which could not have taken part in the ponderomotive action. The correction is, however, neglected as being small in comparison with the combined error of the various measurements. An idea of the magnitude of this neglected correction, as compared with the effect of distributed capacity *et cetera*, could have been obtained by comparing the change in capacity first described with that resulting from the introduction and removal of a strip of much greater vertical width.

(*k*) *Measurement of Frequency.*—Although it was not necessary to measure the frequency, in order to calculate the ponderomotive force, it

was nevertheless of interest to do so, as the values obtained for the dielectric constant would otherwise be meaningless. The wave-meter capacity, let us say C, could be obtained by calibration, and the inductance L, of the loop of copper rod, calculated; but the usual expression for the frequency, n, namely,

$$n = \frac{1}{2\pi\sqrt{LC}}, \tag{17}$$

could not be used for an accurate determination, owing to the fact that the capacity to earth, let us say, x, and, especially, the inductance of the condenser, y, could not be neglected.

If we make a second measurement of the frequency, n, by using a smaller loop, the inductance of which, L_0, may be calculated; at the same time, adjusting the condenser to the proper capacity, C_0, for resonance, we shall have by equating the two expressions corresponding to (17),

$$(L - L_0)x + (C_0 - C)y + C_0L - L_0C = 0. \tag{18}$$

This procedure may be repeaed while the oscillator is emitting waves of some other frequency, n'. The only difference, in this case, will be that the condenser must be set at two new values of capacity. Let us say C' and C_0', instead of C and C_0. We shall then have

$$(L - L_0)x + (C_0' - C')y + C_0'L - L_0C' = 0. \tag{19}$$

Solving (18) and (19) for x and y, we find

$$x = \frac{L_0C - C_0L}{L_0 - L} - \frac{C_0 - C}{L_0 - L}\left(\frac{L(C_0' - C_0) + L_0(C - C')}{(C_0 + C') - (C_0' + C)}\right),$$

$$y = \frac{L(C_0' - C_0) + L_0(C - C')}{(C_0 + C') - (C_0' + C)}.$$

Result.

The writer regrets that, owing to ill-health, it was impossible to make a series of measurements, using various dielectrics, with the degree of accuracy of which the method is capable. It was hoped that at least hard rubber, "Galalith," and black "'Bakelite' Molding Mixture No. 150," might be tried, which substances have dielectric constants, at 3×10^6 cycles per second, of, repectively, 2.15, 2.74, and 3.52.

Fortunately, however, a preliminary measurement was made, with hard rubber as dielectric, which showed the order of magnitude of the force in question. Instead of M_1 and M_2, two other coils were used in this experiment, each 7 cm. in diameter and having 12 turns.

(*a*) *Observed Force.*—The mean deflection was obtained from the mean

of several swings of the torsion balance, first to the right of the zero position and then to the left, by reversing the switch S_2. The shift of the mean position occurred in the expected direction.

The deflections, in cm., were

Right.	Left.
35.6	30.6
33.8	31.5
35.1	30.7
34.6	
Mean.....34.77	30.93
	Difference, 3.84

During the time these observations were being made, the readings of I_1 were 30 and 72 divisions, respectively. The readings of this instrument were found to be proportional to the square of the current; further, the deflection of the torsion balance was, by theory, also proportional to the square of the current; whence both these deflections were proportional to each other. Thus the deflection corresponding to 72 divisions was $[72/(72 + 30)] \times 3.84$, or 2.71 cm.

The constant of torsion, μ, of the fiber used was obtained from the expression for a torsional semi-vibration

$$t = \pi \sqrt{\frac{IL}{\mu}}, \tag{20}$$

where t = the time of a semi-vibration, 450 seconds,

I = moment of inertia of the suspended system, 9,504, and

L = the length of the fiber, 30 cm.;

whence

$$\mu = 1.386.$$

The moment of inertia was obtained from a comparison of the periods of torsional vibration, when two small lead-riders, of equal masses, were placed on the torsion arm, B, Fig. 4, first at one distance, and then at another distance from the axis of suspension. In each case the stress on the fiber, and hence the constant of torsion, was the same. The other quantities involved in the calculation were; arm of the force, 20 cm.; scale distance, 160 cm.

Inasmuch as μ is the couple required to twist unit length of the fiber through unit angle, the actual ponderomotive force, F_m, upon the dielectric was $2.70/(20 \times 30 \times 160) \times 1.386$, or

$$F_m = 0.000391 \text{ dyne.}$$

(*b*) *Calculated Force.*—The expression for the force, F_c, is, from (14)

$$F_c = \frac{p(k-1)(1-\epsilon)}{k} \frac{Hil}{10}, \qquad (21)$$

where p = the proportion of total current flowing through the dielectric strip,

k = the dielectric constant, 2.15,

$(1-\epsilon)$ = the corrective factor, 0.4659,

H = the magnetic field per unit current in the particular coils used in this experiment. The value of H may be taken as 0.3, that for the coils M_1M_2, thus being $0.3 \times 5.23 = 1.569$.

i = the total secondary current as measured by the deflection, 72 divisions of the hot-wire ammeter. By the calibration curve of the instrument, this deflection corresponded to 0.408 ampere.

l = the thickness of the dielectric strip, 0.13 cm.

The frequency was 3.1×10^6 cycles per second.

The value of the calculated ponderomotive force, F_c, was

$$F_c = 0.000732 \text{ dyne.}$$

While it should be remembered that this determination of the force is only approximate, it is nevertheless suggestive that the corrective factor which was neglected, as explained under (*j*), would tend to decrease the calculated value of the force, F_c. Further, the current in the coils M_1M_2 was not so great as that indicated by the hot-wire ammeter, owing to distributed capacity. This, if considered, would also reduce the value of F_c.

In conclusion, I wish to express my gratitude to Deans West and Magie for my opportunity, as research instructor in physics at Princeton University, 1912–1913, of using the facilities of the Palmer Physical Laboratory, where this work was performed. I also wish to thank Professor Webster, of Clark University, for my use, as honorary fellow in physics, of the laboratory of that university, where positive results, concerning the force in question, were first obtained.[1]

SUMMARY.

1. According to Lorentz's theory of a dielectric, a ponderomotive force must act upon such a body when carrying a displacement current in a magnetic field, analogous to the force which is known to act upon a conductor when carrying a conduction current in a magnetic field. An experiment performed by Whitehead, to demonstrate the existence of

[1] Presented in a paper before the American Physical Society, at the Cambridge Meeting, April 27, 1912.

this force, has given a negative result; which result has been shown by others to be in agreement with electrical theory.

2. In the present paper, the theory of a more general research is developed, which includes the Whitehead experiment as a special case, and suggests, further, an experiment by which a positive result may be attained.

3. An apparatus has been designed, on the lines suggested by this theory, whereby the quantities involved in a determination of the force in question may be obtained with a considerable degree of accuracy.

4. An experiment, of a preliminary nature, has shown that the ponderomotive force, upon a piece of hard rubber as dielectric, is of the expected order of magnitude.

WORCESTER, MASS.,
April, 1914.

DETERMINATION OF e/m FROM MEASUREMENTS OF THERMIONIC CURRENTS.[1]

By Saul Dushman.

IN a recent paper on "The Effect of Space Charge and Residual Gases on Thermionic Currents in High Vacuum,"[2] Dr. I. Langmuir has shown that the thermionic current from a heated metallic surface in high vacuum is limited by (*a*) temperature, (*b*) space charge.

With a given potential difference between the electrodes, the current from the heated cathode increases with the temperature in accordance with the equation of Richardson,

$$i = a\sqrt{T}\epsilon^{-\frac{b}{T}}. \tag{1}$$

However, above a certain temperature, this current becomes constant; further increase in temperature does not cause any corresponding increase in thermionic current. The temperature at which this limitation occurs increases with increase in anode potential. It was shown in the above paper that this effect is due to the existence of a space charge produced by the emitted electrons, and it was furthermore deduced that at a fixed cathode temperature, the electron current ought to increase with the three-halves power of the voltage (until the saturation current as defined by the Richardson equation is attained), that is, for electrodes of any shape, the space charge current

$$i = k \cdot V^{3/2}, \tag{2}$$

where V denotes the potential difference, and k is a constant depending on the shape of the electrodes, their area and the distance between them. For the case of the heated filament in a concentric cylindrical anode (infinite length),

$$i = \frac{2\sqrt{2}}{9}\sqrt{\frac{e}{m}} \cdot \frac{V^{3/2}}{r}, \tag{3}$$

where i is the thermionic current emitted per unit length of filament, and r the radius of the anode.

While in the above investigation, a number of results had been ob-

[1] An abstract of this paper was read at the Chicago Meeting, November 29, 1913. See Phys. Rev., *3*, 65, 1914.

[2] Phys. Rev., *2*, 450, 1913.

tained which were in accord with equation (2), it was considered, in view of the skepticism exhibited by many investigators with regard to the actual existence of a pure electron emission that a separate experimental test of the validity of equation (3) would be of interest, especially as it leads to a new and probably very accurate method for the determination of the ratio e/m.

DESCRIPTION OF APPARATUS.

In order to be able to apply equation (3) to the case of a finite length of anode, the latter was inserted between two other anodes on the guard-ring principle. A tungsten filament 15 cm. long and 0.25 mm. diameter was supported by means of a glass tube about 7 cm. in diameter and 36 cm. long (see Fig. 1). The current required to heat the filament was carried by a separate wire at the top, thus avoiding any weakening in the tension exerted by the spring, owing to heating. The anode consisted of three molybdenum cylinders 2.54 cm. in diameter, made from sheet metal about .37 mm. thick. The two edges of the sheet were brought together to form a cylinder and heavy molybdenum strap riveted to the outside of the cylinders, held these edges close together. At the same time, these straps were drilled and tapped to fit 2.5 mm. molybdenum rods. The latter were held rigidly in the side arms by means of molybdenum springs. The outside anodes were each 2.54 cm. long, while the center one was 7.62 cm. long. The connections through the glass were made by means of platinum leads.

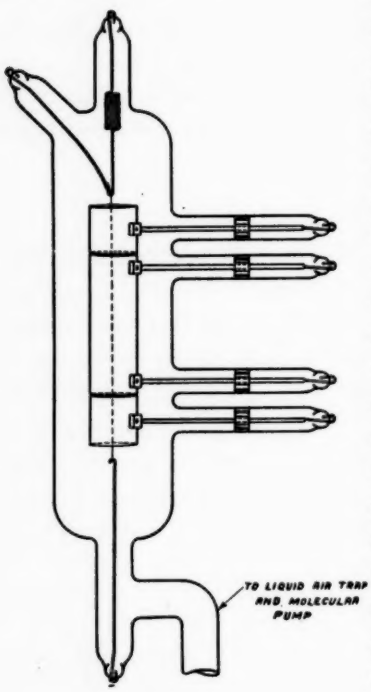

Fig. 1.

The tube shown in Fig. 1 was connected through a large-bore liquid air trap to a Gaede molecular pump. The latter was connected in series with a Gaede "box" oil pump and the rough vacuum line (pressure of about 1 cm.).

A McLeod-gage was inserted between the molecular pump and the oil pump, as well as a tap connected to the atmosphere through a phosphorus pentoxide tube for letting in dry air into the vacuum tube before stopping the molecular pump.

The tube could be exhausted at a temperature of 350° C. by means of an electrically heated oven which was lowered over it.

VACUUM OBTAINABLE.

As stated by Dr. Langmuir in his paper, the most essential conditions for obtaining pure thermionic currents are an extremely high vacuum and absolutely gas-free electrodes.

A special investigation, the results of which will be published shortly in a separate paper, has shown that in order to obtain a very high vacuum even with the molecular pump, it is necessary to insert a liquid air trap between the tube to be exhausted and the pump, and also to heat the tube to over 300° C. for a period of at least one hour. The water-vapor absorbed in the glass walls is thus removed much more quickly and the liquid air trap prevents the diffusion backwards of stopcock grease vapor and other condensible gases. After this preliminary heating, the vacuum is found to be improved considerably. With a pressure of about 0.01 mm. on the rough side of the pump, it is possible in this manner to obtain a vacuum in the tube, which is certainly less than 2×10^{-7} mm.

PREPARATION OF ANODES.

However, under the action of the electrons emitted from the heated cathode, there is a continuous liberation of gas from the anodes, which not only leads to an actual decrease in the thermionic currents themselves, but also prevents the space charge limitation from occurring, owing to the production of positive ionization. It is, therefore, very important to free the anodes thoroughly from occluded gases and volatile oxides which may dissociate gradually when the anodes are heated. This may be accomplished either by heating the anodes to a white heat in a tungsten vacuum furnace,[1] or by powerful electron bombardment in the exhausted tube itself.

The latter method was adopted in this case. A potential varying from 1,000 to 5,000 volts was applied to the electrodes by means of a transformer, the filament being maintained at such a temperature that anywhere from 50 to 200 milliamperes thermionic current was obtained. Under the influence of the electron bombardment at high voltages, the temperature of the anodes was raised to 1000° C. or more and the gases contained in the metal were gradually eliminated. This progressive clean-up of the gases was accompanied by a disappearance of the blue glow, which always indicates the presence of positive ions and by a corresponding increase in the magnitude of the thermionic currents actually

[1] Coolidge,

emitted at any given temperature of the filament. Whenever the blue glow ceased, the voltage was raised, thus heating the anodes to a still higher temperature and liberating more gas from them. By this procedure, the anodes were finally raised nearly to a white heat, and made gas-free.

METHOD OF DETERMINING THE TEMPERATURE OF THE FILAMENT.

The temperatures of the filament were determined from the relation,

$$T = \frac{11{,}230}{7.029 - \log_{10} H},^{1}$$

where H is the intrinsic brilliancy of the filament in international candles per square cm. of projected area, and T denotes the temperatures in degrees Kelvin.

The variation in H with the current carried by the filament was determined by photometric measurements carried out on a special lamp, made up from the same coil of tungsten wire as that used in making up the filament of the discharge tube described above. From these data, a curve was obtained giving the relation between the current carried by the filament and the resulting temperature.

EXPERIMENTS.

In Table I. is given a typical record of observations made on the magnitude of the thermionic currents during the process of evacuation. It will be observed that at the very beginning, when the vacuum was comparatively bad, the electron emission was very low. Replacing the ice and salt mixture around the trap by liquid air caused only a small increase in the thermionic currents obtained. But after the anodes were heated to a high temperature and made fairly gas-free, the increase in the electron emission was considerable. Thus, at 2130° K. the currents observed increased from 0.08 to 2.30 milliamperes, while at 2210° the increase was from 0.1 to 18 milliamperes, and similarly for other temperatures.

These observations are most easily interpreted in terms of Dr. Langmuir's surface film theory. According to this theory, "the effect of gases in changing the electron emission, is due to the formation of unstable compounds on the surface of the wire. In the cases observed (so far), the presence of the compound decreases the electron emission. The extent to which the surface is covered by the compound depends on the *rate of formation* of the compound and on its *rate of removal* from the

[1] I. Langmuir, PHYS. REV., 2, 452, 1913.

TABLE I.

Effect of Gradual Clean-up of Gas in Electrodes.

T = temperature of filament; V = voltage between electrodes; i_1 = thermionic current to center anode only; i_3 = current to all three anodes. Currents measured in milliamperes.

Time.	T	V	i_3	Remarks.
8:00 A.M.				Tube heated in oven to 330° C. for 1 hour.
9:00	2000°	20	0.065	
	2030	20	.14	
	2080	20	.04	*Ice and salt mixture on trap.*
	2130	20	.08	Pressure on rough pump side at about 50 μ. Estimated pressure in tube about 0.001 μ.
	2175	20	.08	Current gradually decreased to 0.04
9:15	2175	50	.04	
	2210	20	.10	
	2210	50	.10	
	2250	50	.22	
	2300	50	.44	
		20	.40	
	2250	50	.26	
	2300	20	.48	
9:30	2315	20	.72	
		50	1.00	Had to stop pump. Pressure on rough pump side still high (50 microns). Cleaned out pump. Put on *liquid air* on trap. Baked out tube in oven for 1 hour.
		95	1.00	
12:00	2180	21	.07	Pressure on rough pump side, 25 μ. Estimated pressure in tube about 0.0005 μ.
	2230	21	.25	
	2330	21	1.1	
12:30 P.M.	2230	21	.38	
	2190	21	.30	
	2230	21	.75	
	2330	21	2.2	Note the gradual increase in thermionic current.
12:45	2250	1,600	1.6	Applied high voltage to bombard anodes.
	2250	2,000	1.2	
	2290	2,000	2.6	Current decreased rapidly to 1.6.
	2330	1,500	8	Current decreased rapidly to 3.
1:00	2330	3,000	40	Current decreased rapidly to 15.
	2365	3,000	90	Current decreased rapidly to 30.
2:00	2400	3,500	130	Current decreased rapidly to 30.
	2360	3,000	70	Current decreased rapidly to 40.
3:00	2400	4,000	140	Current does not decrease so much. No blue glow apparent. Anodes red hot. Changed to low voltage battery.
4:00	2230°	21	12	
	2360	21	14	
	2170	21	9	
	2130	21	3.5	
	2230	21	12	
	2250	28	19	

Time.	T	V	i_3	Remarks.
	2290	28.5	21	
	2160	28	7	
	2160	50	7	Temperature saturation.
	2210	50	18	
	2240	50	27	
	2290	50	48	Space charge effects.
	2330	50.6	49	
			i_1	Measured current to centre anode only.
4:45	2240	50	15.5	See Curve 1, Fig. 2.
	2260	50.2	20	
	2290	50.2	27	
	2310	50.4	28.5	
	2320	50.6	29.0	
	2260	50.2	21	
	2310	50.4	28.5	
		43.4	24.5	
		73.4	32.5	Temperature saturation.
		94.9	32.0	
	2330	95.0	40	
	2345	95.2	55	
	2360	95.3	65	
	2440	96.5	75	
	2460	97.0	77	See Curve 1, Fig. 2.
6:00	2480	97.0	77	Let in dry air.
6:30				Re-exhausted. Fresh liquid air on trap.
7:00	2250	50	10	
	2300	50.4	20	
	2340	50.6	31	
	2390	50.6	34.5	
	2270	50.6	8	
	2270	73	8	
	2310	96	15	
	2340	96	32	
	2390	96	53	
7:30	2440	96	110	See Curve 3, Fig. 2.
			i_3	Applied high voltage to bombard anodes again.
	2340	4,800	66	Current decreased to 40.
	2390	5,000	110	Current decreased to 75.
	2410	5,000	125	Current decreased to 115.
8:00	2450°	5,000	250	Anodes at bright red heat.
	2450	5,000	300	
	2320	5,000	110	
	2380	5,000	225	
	2400	5,000	250	Current does not decrease so much.
			i_1	Measured current to centre anode only at low voltages.
9:00	2440	88	71	

Time.	T	V	i_s	Remarks.
	2400	87	70	See Curve 4, Fig. 2.
	2360	87	68	
	2344	87	60	
	2320	87	50	
	2300	87	38	
	2275	87	28	
	2238	87	17.5	
	2216	87	12.0	
9:30	2350	87	65	
	2380	87	69	
	2450	88	70	Space charge current calculated = 73.
	2200	106	9.5	
	2240	106	20.5	
	2320	106	49	
	2360	107	72	
	2370	107	79	
	2420	108	94	
	2440	108	94	
	2460	108	95	Space charge current calculated = 99.
	2540	108	95	See Curve 4, Fig. 2.

surface." Now even low velocity electrons striking the anode liberate gas from it. If the pump is unable to remove the gas as fast as liberated, the rate of formation of the film on the surface of the cathode exceeds its rate of destruction and consequently the electron emission decreases. This was exhibited strikingly in the actual experiments during the treatment with high voltages. The volume of gas liberated during the interval of only a few minutes was so great that the electron emission would decrease very rapidly.[1] It was only towards the end of the high voltage treatment that the thermionic currents remained fairly steady, thus indicating that little or no gas was being liberated.

The subsequent observations on the thermionic currents at lower voltages show that the vacuum was now considerably better than before the high voltage treatment. Otherwise no space charge effects could have been observed; for as long as gases are given off by the electrodes or the glass walls, there is no indication of space charge effects. In Curve 1, Fig. 2, are plotted the observations obtained between 4:45 and 6:00 P.M. The space charge currents calculated for 50.6 and 97 volts are 32 and 84.5 milliamperes respectively; the values actually obtained were 29 and 77.

The effect of bad vacuum conditions on the thermionic currents is

[1] The high voltage was applied for only a few minutes at a time, until blue glow occurred; the filament was then allowed to cool for a couple of minutes to permit the pump to remove the liberated gases.

shown by the observations made between 7:00 and 7:30 P.M., and plotted in Curves 2 and 3, Fig. 2. After letting in dry air and re-exhausting, it was evidently not possible to obtain a good enough vacuum until the anodes were heated again by high voltage bombardment.

It will be seen from these curves that the effect of the presence of gas is to remove the limitation set by space charge effect, while decreasing the actual electron emission. Curve 3 is interesting because it shows the complete absence of any space charge effects. Evidently the amount of

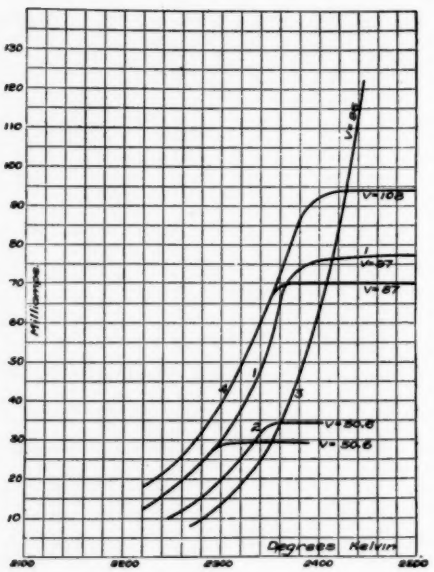

Fig. 2.

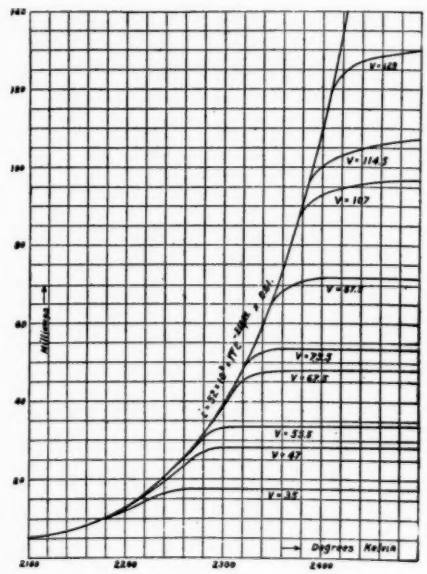

Fig. 3.[1]

gas present in the tube had kept on increasing during these measurements, so that the space charge effect disappeared completely.

After bombarding the anodes again, the results plotted in Curve 4, Fig. 2, were obtained. Owing to the improved vacuum conditions, the electron emission at any temperature was increased, while the space charge currents observed were not very different from those calculated in accordance with equation (3), as shown below.

Table II gives the actually observed space charge currents at different voltages, together with a number of the observations (made under the same conditions) on the total electron emission at different temperatures. The results are plotted in Fig. 3. It is seen that heating the anodes with high voltage again led to no further changes in the values of the thermionic currents observed.

The values under i_1 (calc.) were calculated by means of equation (3)

[1] The ordinates give the current in milliamperes *not* microamperes.

Denoting the constant, $2\sqrt{2/9}\sqrt{e/m}$ by k_0, the values of this constant for different values of e/m are as follows:

$10^{-7} \times e/m$.	$10^3 \times k$.
1.775	14.71
1.766	14.68
1.755	14.62

The last seems to be the most accurate value of e/m, according to the

TABLE II.

Space Charge Currents Obtained at Different Voltages under Best Vacuum Conditions.

Tube exhausted as before. Anodes bombarded with electrons of high velocity by using high voltage, Measured current to center anode only. Pressure on rough pump side, 18 microns. Estimated pressures in tube, less than 0.0004 micron.

T	V	i_1	i_1 (calc).
2125°	47.0	3	
2185	47.0	9	
2236	47.0	18	
2280	47.0	27	
2320	47.0	28	28.5
2340	35	17	17
2300	35	17.3	
2240	35	16.2	
2208	35	12	
2180	35	8.5	
2240	68	19.5	
2262	68	27	
2318	68	45	
2340	68	47	
2400	68	48	49
2340	55.5	34	36
2340	76.0	54	
2400	75.2	54	57.5
2400	88.5	72	
2450	87.5	71	72.5
2480	89	72	74
2480	107.0	96.5	98
2430	108.5	97.7	100
2440	107.5	95.0	
2396	107.5	92.0	
2300	111	49	
2390	114.2	99	
2440	114.5	104.8	
2476	114.8	106.4	
2500	115	107.3	
2522	115	108.0	109
2440	128.5	126	
2480	128.8	128	
2500	129	130	130
2520	129	130	

Treated anodes with high voltage again, and measured thermionic currents to centre, anode only. Pressure on rough pump side down to 10 microns; estimated pressure in tube less than 0.0002 micron.

T	V	i_1	i_1 calc.
2350°	50.8	31.8	32
2180	49	15.5	
2180	70.5	15.0	
2240	71.0	37.5	
2315	71.5	52.2	54
2400	93.5	81.0	80
2490	114.0	111.0	109
2500	136.0	144.0	140

observations of Bucherer and others and is in good accord with the value calculated from the Zeeman effect. It was, therefore, used to calculate the space charge currents for different voltages for the length of filament, corresponding to that of the center anode (7.62 cm.). The results are given in Fig. 4. From this curve, the values given in Table II., under i_1

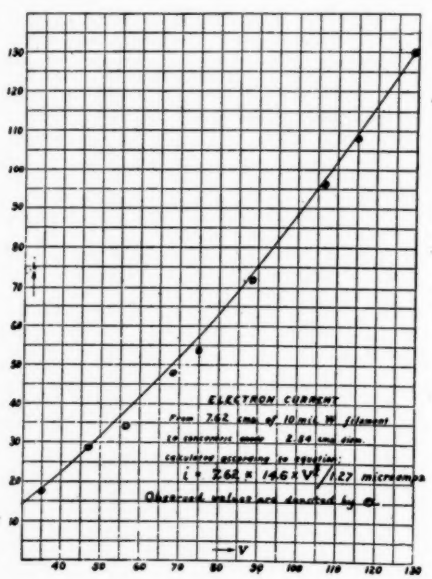

Fig. 4.

(calc.), were taken. The actually observed thermionic currents are also indicated in Fig. 4.

SOURCES OF ERROR.

No attempt was made in this investigation to attain great accuracy; the object being merely to show the validity of the space charge equations.

It may, however, be well to indicate briefly the sources of error involved in the above measurements.

The largest errors undoubtedly occurred in the measurement of V, the potential difference between the electrodes. The voltmeter used was of the ordinary Weston type. As direct current was used to heat the filament, a correction had to be made for the voltage drop through the filament. This drop differed, of course, with the temperature, and was determined by means of a separate set of measurements. To correct for the possibility that the center anode might not be situated symmetrically with respect to the voltage drop through the filament, the thermionic currents were observed for both directions of current through the filament, and the means of pairs of observations are recorded in Table II.

Other sources of error besides that involved in measuring V occurred also in the determination of i_1, the thermionic current to the center anode, and of r, the radius of the cylinder. In neither case was an accuracy greater than 1 to 2 per cent. obtained.

Fig. 5.

Besides these sources of error involved in the actual experimental manipulation there is another factor which must be taken into account when one attempts to use the method for an accurate determination of e/m.

It will be observed that in deriving his space charge formula, Dr. Langmuir intentionally assumed that the electrons emitted from the hot cathode have no initial velocity. This is, however, manifestly not true. Thus the average velocity of electrons emitted from a hot cathode at 3000° K. is about 0.4 volt. In consequence of this initial velocity, the curve giving the potential distribution between anode and cathode will have a form such as that indicated in Fig. 5, where V_0 denotes the potential due to the initial velocity of the electrons, and V, the potential of the anode, both potentials being measured with respect to the minimum point A.

It is therefore evident that instead of using for V in equation (2) the potential difference as measured, we ought to add to the observed value of V another term V_0.[1] Since for small wires, the diameter of the wire does not affect the value of i calculated according to equation (3) we may consider the small distance x_1 which the electrons have to travel before reaching the minimum point as merely an increase in the diameter of the wire and we may therefore neglect it.

[1] It must also be observed that in all the calculations we have assumed an average velocity for the electrons, whereas in the more rigorous calculation Maxwell's distribution law ought to be taken into account.

A more accurate form of equation (3) is therefore:

$$i = \frac{2\sqrt{2}}{6}\sqrt{\frac{e}{m}}\frac{(V_{obs} + V_0)^{3/2}}{r}.^2$$

As in all the experiments recorded above, V_0 was less than 1 per cent. of the observed potential difference it was neglected.

There is no doubt, however, that by carrying out these measurements under more exact conditions the method described in this paper could be used to determine e/m with a very high degree of accuracy.

EFFECT OF GASES.

As has been pointed out by Dr. Langmuir the effect of gases is two-fold. Firstly, it tends to eliminate space charge effects owing to the presence of positive ionization; secondly, the actual thermionic emission is decreased below the value obtained in a good vacuum.

It has already been pointed out that such effects were observed during the process of evacuation. In Table III. are given some data obtained at

TABLE III.

Effect of Nitrogen on Thermionic Currents.

P Rough Side of Pump.	T	V	i_R	i_S	i_0	Remarks.
200 microns	2300°	15	39	9	28	P in tube estimated at 0.01 micron.
		80	39	63	28	
1,000 microns	2325	100	51	86	30	
		80		63	25	
		60		41	18	
		40		22	18	
	2400	79	109	61	85	
		58		39	70	
		38		21	57	
		15		9	20	
	2450	22.5	180	9	75	
		30		14	68	
		52.5		32	75	Blue glow.
		72.5		54	87	Blue glow.
		93.5		79	105	Blue glow.

the end of an experiment with gas-free electrodes in presence of nitrogen. Nitrogen of the same degree of purity used in the manufacture of the nitrogen-filled tungsten lamps was allowed to enter the discharge tube

[2] The effect of taking into account the initial velocity of the electrons and Maxwell's distribution law has been discussed very fully by W. Schottky in a recent paper (Physik. Zeitsch. *15*, 624) who has also pointed out that in making use of the space charge equation for an accurate determination of e/m, failure to consider the above factors might involve errors of over 2 per cent. (Physik. Zeitsch. *15*, 528, footnote.)

through a narrow-bore stopcock connection sealed between the liquid air trap and the pump. The rate of flow of the gas was measured approximately by noting the pressure on the rough pump side. From this reading it was possible to obtain an estimate of the pressure in the tube.

Under i_r is given the value of the electron emission at the corresponding temperature as obtained from the curve $i = a\sqrt{T}\epsilon^{-\frac{b}{T}}$ given in Fig. 3. The values under i_s are the space charge currents calculated for the corresponding voltages by means of equation (3) and plotted in Fig. 4. The actually observed thermionic currents are given under i_0. It will be observed that *even with blue glow* the thermionic currents obtained were much lower than those calculated from the Richardson equation.

This decrease in thermionic currents was certainly not due to space charge effects, as is seen from the data at $T = 2325$. In some other experiments in which air was allowed to enter the tube, the decrease in thermionic current was so pronounced that even with 3,000 volts on the anode the sudden appearance of gas in the tube (whether originating in the anode or allowed to flow in through a side connection) immediately caused the electron emission to fall to almost zero.

GENERAL CONCLUSIONS.

The above experiments are thus in uniform agreement with the observations and explanation given by Dr. Langmuir in his paper on space charge effects, and in the light of all our subsequent experiments his conclusions on the nature of electron emission from heated metals appear to be well justified. The *perfect definiteness* of the results obtained which are independent of vacuum conditions after a sufficiently high vacuum has once been attained, the *reproducibility* of the observations even after allowing gas to enter the tube and then re-exhausting, and the quantitative agreement obtained not only in the above experiments but in all the experiments so far carried out in this laboratory point to the existence of a *pure electron emission per ipse*, which is not a secondary effect due to chemical reactions, as assumed by a number of other investigators, and which is a *function of the temperature only*.

In conclusion the writer wishes to express his indebtedness to Dr. I. Langmuir for kindly suggestions and interest during the progress of the above investigations.

SUMMARY.

The space charge formula developed by Dr. I. Langmuir for the thermionic current from a heated filament to a coaxial cylindrical anode has been tested experimentally over a range of voltages from 35 to 140, for

the case of a tungsten filament in a concentric molybdenum cylinder.

The results obtained are in good agreement with this formula and indicate that the method ought to prove a very accurate one for the determination of e/m. The observations obtained on the effect of gases on the thermionic currents are also found to be in accord with Dr. Langmuir's surface film theory and justify the conclusion that there exists a pure electron emission from heated metals which is a function of the temperature only and is not a secondary effect due to presence of gases as assumed by a large number of investigators.

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THE EFFECT OF A LONGITUDINAL MAGNETIC FIELD ON SPARK POTENTIALS.

By Robert F. Earhart.

THE present paper may be regarded as a continuation of an article published in the February number of this journal on the "Discharge in a Magnetic Field."[1] The former paper sought to extend the work of Paalzow and Neison[2] by studying the variation in the magnitude of a discharge current between parallel plate electrodes. The electric force and magnetic force were parallel. The quantitative measurements showed that for gas pressures exceeding the critical pressure, the effect of a magnetic field was to diminish the current. As the pressure was reduced the effect of the field diminished and as the critical value was approached the effect vanished. For pressures less than the critical pressure small fields increased the current strength but strong fields were not so effective as the lesser ones in some cases.

The effect of the weaker fields in increasing the current was explained by Townsend[3] in commenting on some experiments of Strutt.[4] He suggests that the helical motion imparted to an electron by the longitudinal field will so increase its path that ionization by collision will take place with a smaller potential gradient between the electrodes.

For higher pressures it would seem that this would be offset by another effect. In a luminous discharge at the higher pressures the motion of an ion or electron will probably be complex because collision is frequent and the forces on any electrified unit will vary continually in magnitude and direction. If this be true the motion of an ion will at times have a component velocity normal to the lines of force and thus be subject to a force tending to divert it to the walls of the tube. If a longer tube of the same section should be substituted the opportunity for the charged body to be diverted would be increased. Again if the tube should be maintained of constant length but its section increased together with the area of the electrodes the probability of the ion reaching the electrodes would be increased.

[1] Phys. Rev., N. S., Vol. III., p. 103, 1914.
[2] Wied. Ann., I.–XIII., p. 207, 1897.
[3] Phil. Mag., VI., 26, p. 730, Oct., 1913.
[4] Proc. Roy. Soc., *A*, 89, p. 68, 1913.

Having this in mind it seemed possible to attempt a test of the hypothesis. A discharge chamber D in Fig. 1 was placed between two brass electrodes A and B. These were mounted on the poles of a large electromagnet by means of fiber collars. The pole pieces M, M in the figure were 5.08 cm. across the face so that the midportion of the field was quite uniform. This was tested and with satisfactory results. Fields of 10,000 C.G.S. could be obtained with the gap made necessary by the insertion of the discharge chamber. The spark chamber was made from a disk of hard rubber sealed to the electrodes. The disk itself served to measure the distance between the electrodes and at the same time confine the discharge to the uniform portion of the electric and magnetic fields.

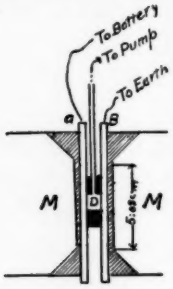

Fig. 1.

The length of the discharge path was made 2.5 mm., 5.0 mm. and 10.0 mm. and in each case apertures of three sizes were used. The apertures were 3, 6 and 9 mm. approximate radius. They were bored out with a $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$ inch drill. The ratio of their areas is therefore 1, 4, 9. The electrical arrangement was similar to the one described in the previous paper and is not reproduced. In each of

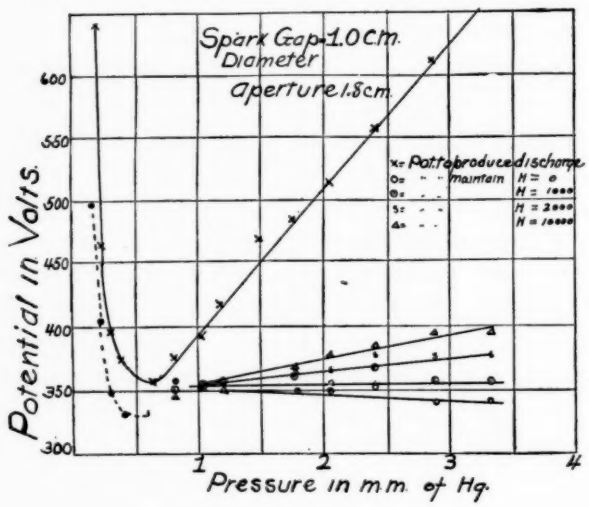

Fig. 2.

the nine cases the gas pressure was varied between .2 mm. and 5 to 6 mm. The fields were of strength 0, 500, 1,000, 2,000, 5,000, 10,000 C.G.S. units. Air was the only gas used.

It is well known that for a particular pressure a definite potential is required to start a discharge but that once started a lower potential will maintain it.

All these experiments show that for pressures above the critical value the potential required to start the discharge is unaffected by the magnetic field but that the potential required to maintain the current flow must have a higher value when a field is on than for a zero field. This is shown in figure 2, where the distance separating the electrodes is 1 cm. and the aperture is 18 mm. in diameter. The variations in the potential required to maintain the discharge are quite appreciable. This is in

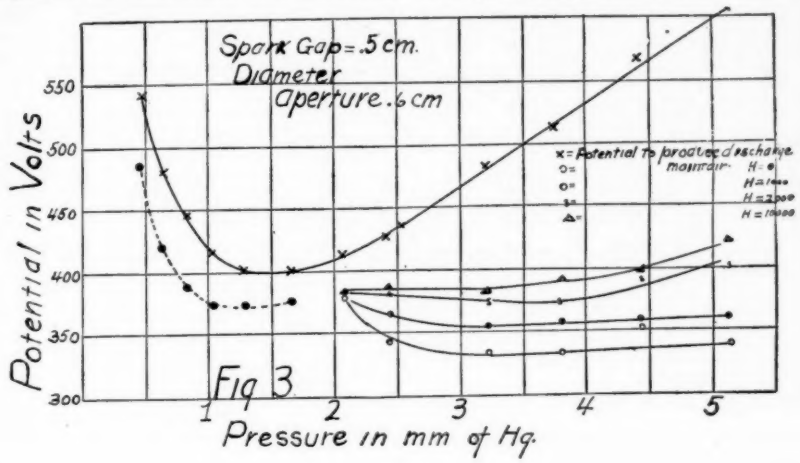

Fig. 3.

agreement with the current-potential curves obtained in the previous experiment because the current value depends on the excess of potential over the value required to maintain the current rather than its relation to the potential required to start the discharge. The graphs show only the results for fields of strength 0, 1,000, 2,000, and 10,000 units. It will be noted that these curves converge near the critical pressure. For pressures below the critical value the discharge potential is reduced by the magnetic field. These are shown by full black circles. Here the variations due to different fields are small. The ones represented are for a field of 10,000 units. The values for the fields from 1,000 to 10,000 differ very slightly from the ones shown. Figs. 3, 4 and 5 exhibit the variation due to size of aperture. It will be noted that a higher potential is required to maintain the discharge for the smaller aperture. Fig. 5 should be compared with Fig. 2 to show the effect of length of path. All of the nine families of curves (four only are reproduced) are consistent with one another save a vagary exhibited by the tube 1 cm. long having the smallest aperture. In that case all the effects due to variations of the field were duplicated but the entire family of curves was displaced upward along the *Y*-axis, meaning that higher potentials were required.

In my judgment this is due to another effect superposed on the one sought. In a paper entitled "Some Characteristic Curves for Gases at Low Pressures"[1] the author reported on some experiments made when

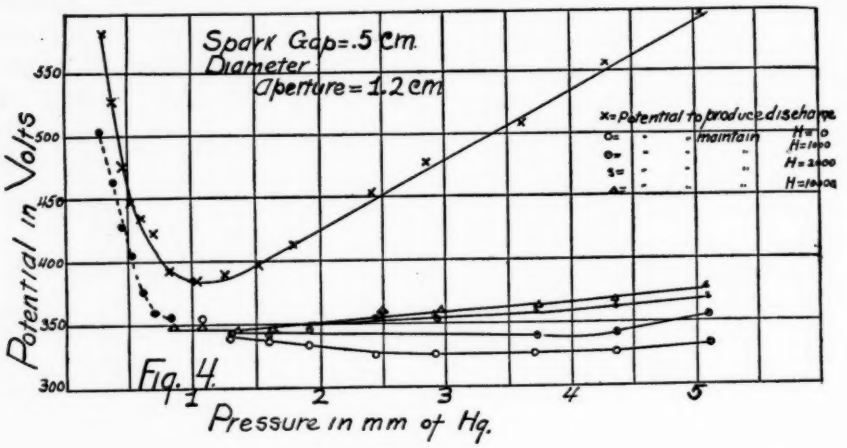

Fig. 4.

thin diaphragms having small apertures were interposed between the electrodes, also when the discharge was made to pass through a tube serving as a constriction in the path. It was found that the effect of

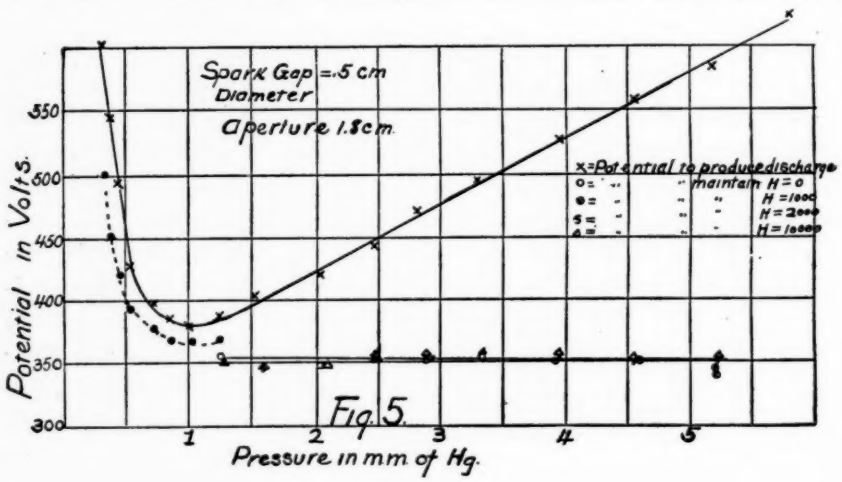

Fig. 5.

introducing such a constriction was determined by the length of the constricted portion and that the potential required to maintain a discharge of given strength had to be raised. The only effect on the characteristic

[1] PHYS. REV., N. S., Vol. I., p. 85, February, 1913.

curve thus obtained was to displace it upward along the potential axis. It seems that in the case of the long tube with the smaller aperture that the superposition of constrictive effect on the other is to require a higher potential to produce the same variations of change due to the magnetic field.

The conclusions are:

1. Spark potentials for pressures above the critical pressure are unaffected by a longitudinal field.

2. Spark potentials below the critical pressure are decreased by a longitudinal field.

3. When the pressure exceeds the critical one, the existence of a longitudinal field requires a higher potential to maintain the discharges than if no field existed.

4. The variations noted in (3) are increased with increasing fields and become larger as the gas pressure is increased.

PHYSICAL LABORATORY,
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COLUMBUS, OHIO,
April 20, 1914.

ON THE PASSAGE OF CATHODE PARTICLES THROUGH GASES AT LOW PRESSURE.

BY L. W. MCKEEHAN.

THE work to be described was undertaken primarily to determine whether a very narrow pencil of cathode rays is scattered by traversing a high vacuum. Such an effect may be expected if the electric field of each electron in the atoms composing the walls of the vessel or outside it, is confined to a few narrow tubes or filaments extending to infinity. This structure of the electric field was suggested by Sir J. J. Thomson in his Electricity and Matter (p. 63, 1903).

The cathode rays were obtained from a plane aluminium cathode in a spherical bulb about 15 cm. in diameter. They passed into the test chamber through a cylindrical brass tube 5 cm. long and of .02 cm. internal diameter set in a plane anode directly opposite the cathode. The test chamber was about 60 cm. in length and 3 cm. in diameter, and terminated in a transverse tube of larger diameter that served as a photographic camera. All glass to glass, or glass to brass joints were sealed with sealing wax. Diffusion through the hole in the anode was so slow that a good vacuum could be kept in the test chamber without lowering the pressure in the discharge tube to such a value that the photographic effect of the rays would be much reduced. The attainment of exceedingly high vacua was not attempted, since at pressures easily reached by liquid air and cocoanut charcoal (5×10^{-5} cm. of mercury by a MacLeod gauge of about one liter capacity) no scattering was detected.

Proper precautions to prevent any electrostatic fields, or any except axial magnetic fields, were taken by using a brass lining within the test chamber, by cleaning all metal surfaces exposed to rays, and by placing the axis of the apparatus parallel to the magnetic field due to the earth and to the induction coil.

After leaving the hole in the anode the narrow cone of rays traversed about 30 cm. before striking a small aperture or solid obstacle, and the rays not stopped traversed about 30 cm. more before reaching the photographic plate. A series of exposures on the same plate could be taken by the method used by Sir J. J. Thomson for positive rays.[1]

[1] Phil. Mag., VI., p. 228, 1911.

The absence of scattering in a high vacuum was shown by the fact that the shadows cast on the photographic plate were geometrically obtainable by drawing straight lines enveloping the obstructions from a point on the axis of the apparatus, which, in case the pressure in the discharge tube was fairly high, was within the anode, but which was near the cathode, in case the pressure in the discharge tube was low enough to make the scattering due to the gas unimportant. The accuracy with which measurements could be made on the photographs was limited by the width of the transition from fully exposed to unaffected areas, about .002 cm. The sharpness of this transition proved that the rays were not uniformly distributed across the aperture in the anode, but were principally axial, as was expected from the necessary obliquity of the electric field close to the anode except along the axis of the aperture.

From the dimensions of the apparatus it was clear that an average angle of scattering exceeding 30 seconds of arc would have been detected with certainty if it had occurred. One series of photographs with rays of varying velocity passing through a slit .029 cm. wide is shown in Fig. 1, exposures 1, 3, 5, 7, and 9, on the center line of the figure. Fig. 2 shows the shadow cast by a cross of copper wire placed in the path of the rays. In exposures 1, 2, and 3 the cross was connected to the anode, which was always to earth, and the geometrical shadow is obtained. In the other exposures the cross was disconnected from the anode, and received a negative charge from the rays which it stopped. These latter photographs (one of which has been intensified in the print) show the shape of the field so clearly that they suggest a method for measuring the electric intensity on a convex surface in a high vacuum.

The rays were not homogeneous in velocity but most of them were of approximately the same speed, as shown in Fig. 1, exposures 1, 3, 5, 7, and 9, and in Fig. 2, where the more diffuse spots at one side of the center line are produced by magnetic deflection between the slit and the photographic plate. The coil for this purpose consisted of a few turns of wire wound on a rectangular frame with semicircular ends. This was placed outside the test chamber with the axes of the two coinciding. If such a coil of length $2l$ and of width a is placed with its center at a distance d from a source of axial cathode rays of velocity v and corresponding e/m the deflection x on a photographic plate at a distance from the source equal to $2d$ corresponding to ni ampere-turns is given to terms of the second order by the equation

$$x = \frac{4nie}{mv} \cdot \frac{d}{a}\left[\frac{(l+d)^2}{\{a^2+(l+d)^2\}^{\frac{1}{2}}} - \frac{(l-d)^2}{\{a^2+(l-d)^2\}^{\frac{1}{2}}}\right].$$

The average velocity of the fastest group of rays used, measured in this way, varied from 2.7×10^9 to 8.0×10^9 cm./sec.

Upon raising the pressure in the test chamber the scattering due to the gas became apparent in a reduction of intensity in the photographs at pressures of air as low as 3×10^{-4} cm. of mercury, but even at 1.2×10^{-3} cm., the highest pressure at which photographs were obtainable with an exposure of one hour or less in this apparatus, the geometrical pattern obtained was of the same size, and as clear cut, as at the lower pressures, although very faint, and overlaid by scattered radiation extending with decrease of intensity to more than a centimeter from the center of the photograph. The conclusion to be drawn is that at this pressure a considerable number of these electrons (velocity about 5×10^9 cm./sec.). have free paths in excess of 60 cm., or more than 100 times the mean free path of a molecule of the gas.

Recent experimental work on slow cathode rays by J. Franck and G. Hertz[1] indicates that for such rays the mean free path is $4\sqrt{2}$ times that of a gas molecule, that is, for the case discussed, 3.4 cm. Extrapolation from the values given by Townsend[2] for negative ions of sub-atomic dimensions gives 6 cm. as the mean path between ionizing collisions under the same set of conditions. The still higher value indicated by these experiments suggests that the high-speed cathode rays must, in many cases, pass completely through the atom without measurable change of direction.

Positive rays could be obtained without alteration of pressure by reversing the discharge current. In Fig. 1, exposures 2, 4, 6 and 8, taken in this way show that the positive rays are much scattered at a pressure that does not affect the cathode rays. The atomic size of the positive rays explains the greater chance of a collision and consequent deflection.

An attempt was made to deduce the law of scattering of cathode rays from the variation of intensity with distance from the center of the photograph, using approximately circular apertures to limit the beam of rays after sorting them magnetically with a solenoid. The hole in the anode was made several millimeters in diameter to permit a greatly increased number of rays to be dealt with, and since discharge tube and test chamber could not be kept at different pressures the average velocity of the rays was altered by Whiddington's method.[3] Conditions were held constant for longer periods of time by supplying gas through a very fine capillary tube and removing it continuously with a Gaede mercury pump.

[1] Verh. d. D. Phys. Ges., 15, p. 34, 373, 929 (1913); 16, p. 12 (1914).

[2] Ionization by Collision, p. 30 (1910).

[3] Camb. Phil. Soc. Proc., 17, p. 251, 1913.

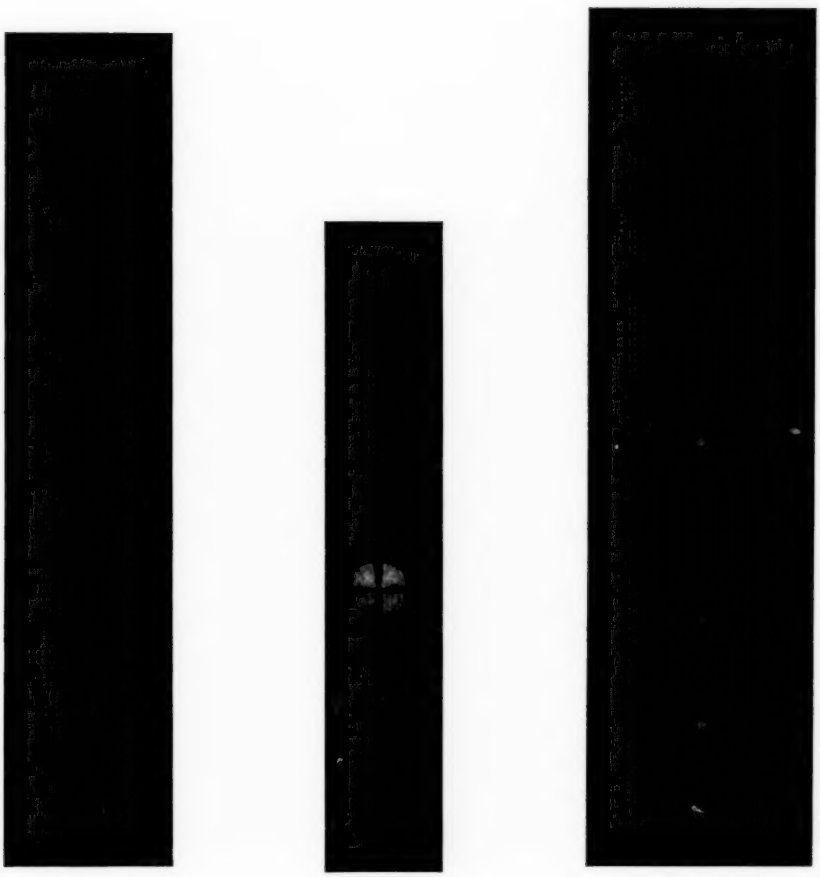

FIG. 1. FIG. 2. FIG. 3.

L. W. MCKEEHAN.

In each experiment a series of exposures for widely differing times was obtained under conditions kept as nearly constant as possible, but in different experiments the nature of the gas, its pressure, and the velocity of the rays were varied independently. Fig. 3 shows the photographs obtained in oxygen at 4.56×10^{-4} cm. pressure with rays of velocity 6.5×10^{9} cm./sec. The time of exposure varies from five seconds to one hour.

If the photographic intensity were directly proportional to the number of rays received, a single exposure on each plate would suffice to give the relation between distance from the center and number of rays. This was not the case as experiment showed, for the ratio of intensities at two given distances from the center was not the same on the different exposures. However, if conditions are really constant during an experiment, and if the intensity of the photograph on a small area of the plate depends only on the number of rays of approximately the same velocity received there, although not proportional to it, the law of scattering for a given gas atom and for a given velocity of rays can be found by measuring the distances from the centers of the several exposures in the appropriate experiment to circular zones in which the photographic intensity is identical. Each pair of unequally timed exposures gives the relation between distance from the center and relative number of cathode rays, and all pairs on the same plate should give the same relation. As long as the greater part of the pencil of rays is not diffused, the scattering is single and not compound[1] since the probability of more than one deflection is small.

In the experiments actually performed in this way consistent results were not obtained. The chief reason for this failure lies in the wandering of the beam of rays within the discharge tube, due to surface charges and fluctuations of discharge potential. These alterations of direction were directly observed on fluorescent screens placed within the test chamber, and were so considerable as to vary the intensity of the photograph by as much as one half in consecutive exposures of several minutes duration.

Another difficulty inherent in the method is the small amount of scattered radiation that can be obtained within the limits of single scattering, and the consequent faintness of the photographs. This faintness enhances errors due to fogging by light or by X-rays, to original non-uniformity of the sensibility at different points on the plate, to non-uniform development, and to reversion of the latent image. The layer of the emulsion affected by homogeneous cathode rays is so thin

[1] Sir Ernest Rutherford, Phil. Mag., VI., 21, p. 669, 1911.

that inversion began at low intensity, a second inversion took place at the center in all long exposures, and a third inversion was just detectable in one photograph. The use of sensitized paper in place of dry plates was found to overcome the last mentioned defect, but no better results were obtained.

One of the assumptions made above is open to attack on theoretical grounds. Scattered rays should have smaller velocities than unscattered, and the photographic effect should not therefore be quite proportional to number of rays, but should depend slightly on the angle of deflection. It can be seen that this effect should be small for small angles, but it is inseparable from the photographic method.

A consideration of the number of rays available, and of their ionizing power at the necessary low pressure made it seem inadvisable to attempt an electrical method of counting, especially since the chief defect of the experiments would not be overcome by such a change. The general problem of the deflection of an electron by matter is being attacked in this laboratory by a more direct and powerful method,[1] from which definite results are expected.

In conclusion thanks are due to Sir J. J. Thomson for the suggestion of the original problem, and for the facilities afforded during that part of the work performed in the Cavendish Laboratory.

PHYSICAL LABORATORY,
UNIVERSITY OF MINNESOTA,
April 27, 1914.

[1] A. F. Kovarik and L. W. McKeehan, PHYS. REV., II, 3, p. 149, 1914.

THE SPECIFIC HEAT OF SOLIDS AND THE PRINCIPLE OF SIMILITUDE.

BY RICHARD CHACE TOLMAN.

THE writer has already presented[1] a number of derivations in widely different fields of physical science, all of which are based on a single postulate which may be called the *principle of similitude.* In the present article we shall make use of this same principle in deriving a formula for the specific heat of homogeneous, isotropic, elastic substances.

We shall find that the formula thus obtained for specific heat agrees with that of Debye[2] for the specific heat of solids and liquids at low temperatures. In our proof, however, it will *not* be necessary to make use of any of the various forms of quantum theory. In particular, the derivation of Debye rests on the special assumption borrowed from the quantum theory of Planck that

$$E_\nu = \frac{h\nu}{\epsilon^{\frac{h\nu}{kT}} - 1} d\nu$$

is the energy in an elastic sphere at temperature T associated with modes of vibration whose frequencies lie between ν and $\nu + d\nu$. In our derivation, however, *we shall make no assumptions as to the form of the relation connecting energy with temperature and frequency of vibration.*

This possibility of deriving a satisfactory formula for the specific heat of solids and liquids without the special assumptions of the quantum theory is of particular interest at the present time, since in spite of brilliant justification from the experimental standpoint, the theoretical foundation of the quantum theory must still be regarded as a field where further research is necessary.

It is also interesting to point out that the derivation of the Debye formula which we shall present will necessitate no new investigations in the field of elasticity theory, but will merely make use of the well-known expressions for the velocity of wave motions in elastic solids.

PART I. THE PRINCIPLE OF SIMILITUDE.

The principle of similitude may be stated as follows:—*The fundamental entities of which the physical universe is constructed are of such a nature*

[1] Tolman, PHYS. REV., *3*, 244 (1914).

[2] Debye, Ann. d. Physik, *39*, 789 (1912).

that from them a miniature universe could be constructed exactly similar in every respect to the present universe. For a discussion of this new postulate we must refer the reader to the article already mentioned, and shall here confine ourselves merely to a recapitulation of those deductions from the principle which are necessary for our immediate purpose.

Let us consider two observers, O and O', provided with instruments for making physical measurements. O is provided with ordinary meter sticks, clocks and other measuring apparatus of the kind and size which we now possess, so that he can duplicate all our measurements in the present physical universe. O', on the other hand, is provided with a shorter meter stick, and correspondingly altered clocks and other apparatus so that he can make measurements in the miniature universe of which we have spoken and in accordance with our postulate obtain exactly the same numerical results in all his experiments as does O in the analogous measurements made in the real universe.

If now O and O', instead of each remaining in his own universe, should both make measurements of the same physical objects and occurrences, it is evident that they would not obtain the same results, since their units of measurement are different. Let us suppose that O and O' both measure the *same* length, then if O's meter stick is x times as long as that of O', it is evident that their measurements of the given length would be connected by the equation $l' = xl$.

In the article already referred to it was found possible to obtain a whole series of transformation equations of the type $l' = xl$, connecting the measurements made by O and O' of given lengths, time, intervals, masses, forces, charges, etc. These relations were derived by combining with the postulate of similitude, certain principles taken from the Einstein theory of the relativity of motion and from the electron theory. We shall now present for future reference a table of such transformation equations.

TABLE OF TRANSFORMATION EQUATIONS.

For length	$l' = xl$	(1)
For area	$S' = x^2S$	(2)
For volume	$V' = x^3V$	(3)
For an interval of time	$t' = xt$	(4)
For frequency	$\nu' = \nu/x$	(5)
For velocity	$v' = v$	(6)
For acceleration	$a' = a/x$	(7)
For quantity of electricity	$e' = e$	(8)
For mass	$m' = m/x$	(9)
For density	$\rho' = \rho/x^4$	(10)
For force	$f' = f/x^2$	(11)
For pressure	$P' = P/x^4$	(12)
For energy	$E' = E/x$	(13)

For energy density..... $u' = u/x^4$ (14)
For absolute temperature..... $T' = T/x$ (15)
For the coefficient of compressibility..... $\chi' = x^4\chi$ (16)[1]
For Poisson's Ratio..... $\sigma' = \sigma$ (17)[1]

[1] Of these transformation equations the only ones not already derived in the article referred to are those for the coefficient of compressibility and for Poisson's ratio. The coefficient of compressibility may be defined by the equation

$$\chi = \frac{1}{V}\frac{dV}{dP},$$

while Poisson's ratio, σ, is that between the longitudinal and transverse strain of an elastic substance. The derivation of the transformation equations for these two quantities is obviously merely a matter of substituting equations already given above.

PART II. APPLICATION OF THE PRINCIPLE OF SIMILITUDE TO DETERMINE THE FORM OF THE FUNCTION.

The general method of applying the principle of similitude to determine the form of functional relations connecting physical magnitudes is to consider some construct which could exist either in the actual universe or in the miniature universe which to observer O' appears the same as the actual universe. It is evident from the principle of similitude that the properties of a construct which has the same general character in both universes will have to obey the same general laws, whether measured by observer O or by observer O', while a further condition will be imposed upon the magnitude of these measurable properties by the transformation equations which we have just developed. These two sets of conditions will permit the attainment of definite information as to the necessary form of the functional relation connecting the measurements of different properties of the construct.

In the article already mentioned this method was used for deriving the law of Charles for an ideal gas, for showing that the specific heat of such an ideal gas would be independent of the temperature, for deriving Stefan's law for the energy density of a hohlraum, for obtaining a relation between energy density and frequency in a hohlraum, and for deriving various relations in the fields of electromagnetic and gravitational theory. We may now apply the method to obtain information as to the form of the functional relation connecting the energy density of an elastic substance with its temperature and elastic constants.

In the first place it is evident that a given homogeneous, isotropic, elastic substance is a construct which would appear to be homogeneous, isotropic and elastic both to observer O and to observer O', and hence by the principle of similitude experiments made on it by either observer would have to lead to the same general laws. It is also evident that the energy density u, of such a substance will be completely determined by

its temperature T and the three constants—density ρ, compressibility χ, and Poisson's ratio σ—which completely determine its elastic properties.[1] Suppose now the measurements of O have led to the following relation, between the energy density and the elastic properties and temperature of the substance,

$$u = F(\rho, \chi, \sigma, T) \tag{18}$$

then by the principle of similitude the measurements of O' will have to lead to a similar relation,

$$u' = F(\rho', \chi', \sigma', T') \tag{19}$$

where F must have the same form as in the previous equation.

We have, however, already found transformation equations, Nos. (14), (15), (10), (16), and (17), connecting the measurements of energy density, temperature and elastic constants made by O' with those made by O. Substituting in equation (19) we have

$$\frac{u}{x^4} = F\left(\frac{\rho}{x^4}, x^4\chi, \sigma, \frac{T}{x}\right)$$

or solving for u and combining with (18) we obtain,

$$u = x^4 F\left(\frac{\rho}{x^4}, x^4\chi, \sigma, \frac{T}{x}\right) = F(\rho, \chi, \sigma, T) \tag{20}$$

Since the total energy of an elastic body will be proportional to its volume we may write,

$$E = Vx^4 F\left(\frac{\rho}{x^4}, x^4\chi, \sigma, \frac{T}{x}\right) = VF(\rho, \chi, \sigma, T) \tag{21}$$

Since x may be any number we have thus obtained from the principle of similitude an equation which imposes very definite conditions on the form of the functional relation connecting the energy content of an elastic body with its temperature and elastic properties.

By considering some familiar results of the theory of elasticity we may complete the solution of our problem.

PART III. APPLICATION OF ELASTICITY THEORY.

In the previous section we have derived from the principle of similitude a functional equation, No. 21, which presents certain necessary relations between the energy content of an elastic body and its volume, elastic constants, and temperature. Let us now see what conclusions we can draw from the *theory of elasticity* as to the energy content of an elastic body.

[1] We shall assume a substance in which the elastic properties are independent of the temperature.

Let us consider an elastic body of some definite shape and size, fixed at its outer surface. At a given temperature such a body will contain a certain amount of energy and we may think of this energy as due to the presence of systems of standing waves which undergo continual reflection at the fixed boundary. These waves may either transverse or longitudinal. In the former case the theory of elasticity has shown that their velocity will be $\sqrt{\mu/\rho}$ and in the latter case $\sqrt{(\lambda + 2\mu)/\rho}$, where ρ is the density of the material and μ and λ its elastic constants. These constants are connected with the measurable quantities compressibility, χ, and Poisson's ratio, σ, by the equations

$$\mu = \frac{1}{\chi}\frac{3(1-2\sigma)}{2(1+\sigma)} \tag{22}$$

and

$$\lambda = \frac{1}{\chi}\frac{3\sigma}{1+\sigma}. \tag{23}$$

If now we should change the dimensions of the body or its elastic properties, it is evident that we should also change the frequency of the standing waves which it contains at thermal equilibrium.

If we consider changes in the *volume* of the body without alteration of shape, it is evident that the linear dimensions of the body will always be proportional to the cube root of the volume, $V^{\frac{1}{3}}$. Hence the wave lengths of the different standing waves which the body contains will be proportional to the cube root of the volume or their frequencies to its reciprocal.

On the other hand, if we change the *elastic properties* of the body we should also change the frequencies of the different standing waves since we change the velocity of propagation. For transverse waves the frequencies will be proportional to the velocity of transverse waves, $\sqrt{\mu/\rho}$; and similarly for longitudinal waves the frequencies will be proportional to $\sqrt{(\lambda + 2\mu)/\rho}$.

Hence we may write for the frequencies of the different standing waves which a body of given shape will contain

$$\nu_1 = \frac{a_1}{V^{1/3}}\sqrt{\frac{\mu}{\rho}},\ \nu_2 = \frac{a_2}{V^{1/3}}\sqrt{\frac{\mu}{\rho}},\ \nu_3 = \frac{a_3}{V^{1/3}}\sqrt{\frac{\mu}{\rho}},\ \text{etc.},$$

$$\nu_1' = \frac{a_1'}{V^{1/3}}\sqrt{\frac{\lambda+2\mu}{\rho}},\ \nu_2' = \frac{a_2'}{V^{1/3}}\sqrt{\frac{\lambda+2\mu}{\rho}},\ \nu_3' = \frac{a_3'}{V^{1/3}}\sqrt{\frac{\lambda+2\mu}{\rho}}\ \text{etc.}, \tag{24}$$

where ν_1, ν_2, ν_3, etc., are the frequencies of the various transverse, and ν_1', ν_2', ν_3', etc., are the frequencies of the various longitudinal standing waves. a_1, a_2, a_3, etc., and a_1', a_2', a_3', etc., are constants which depend

on the shape of the body but are independent of its volume or elastic properties.

Now let us assume that at thermal equilibrium the average energy associated with each loop of a standing wave is determined merely by its frequency and the temperature. Then in the case of our elastic body we may write for the energy associated with a standing wave of frequency ν,

$$E_\nu = nF'(\nu, T)$$

where n is the number of loops in the standing wave under consideration, and F' is the unknown functional relation connecting energy with frequency and temperature.

For the total energy of the body we may write

$$E = \Sigma E_\nu = \Sigma nF'(\nu, T)$$

where the summation Σ is to include all the different waves in the body.

We have already seen, however (equation 24), that the frequencies of the different waves are determined by equations either of the form

$$\nu = \frac{a}{V^{1/3}}\sqrt{\frac{\mu}{\rho}}$$

or

$$\nu = \frac{a'}{V^{1/3}}\sqrt{\frac{\lambda + 2\mu}{\rho}},$$

where a and a' are constants depending on the shape of the body. Hence we may write for the total energy of the body

$$E = \Sigma E_\nu = \Sigma nF'(\nu, T) = \Sigma nF'\left(\left[\frac{a}{V^{1/3}}\sqrt{\frac{\mu}{\rho}}\right], T\right) + \Sigma' n'F'\left(\left[\frac{a'}{V^{1/3}}\sqrt{\frac{\lambda+2\mu}{\rho}}, T\right]\right), \quad (25)$$

where the summation Σ includes all the transverse waves and Σ' includes all the longitudinal waves. In this equation n, n', a and a' are constants independent of the volume or elastic properties of the body.

Let us now transform this equation by substituting for μ and λ their values in terms of compressibility and Poisson's ratio as given in equations (22) and (23). We shall obtain

$$E = \Sigma E_\nu = \Sigma nF'(\nu, T) = \Sigma nF'\left(\left[\frac{a}{V^{1/3}}\left(\frac{1}{\rho\chi}\frac{3(1-2\sigma)}{2(1+\sigma)}\right)^{1/2}\right], T\right) + \Sigma' n'F'\left(\left[\frac{a}{V^{1/3}}\left(\frac{1}{\rho\chi}\frac{3(1-\sigma)}{1+\sigma}\right)^{1/2}\right], T\right). \quad (26)$$

We thus have obtained from the theory of elasticity a functional relation

between the energy of an elastic body, and its volume, density, elastic constants χ and σ, and its temperature.

We have already obtained, however, from the principle of similitude another functional relation, equation 21, between these same quantities. This equation was

$$E = Vx^4 F\left(\frac{\rho}{x^4}, x^4\chi, \sigma, \frac{T}{x}\right) = VF(\rho, \chi, \sigma, T).$$

By comparing these two expressions we see since the x's must cancel and since volume must enter to the first power, that $F'([\,], T)$, which occurs in equation (26) must be of the form,[1]

$$E_\nu = k\frac{T^4}{[\,]^3} = k\frac{T^4}{\nu^3}, \tag{27}$$

where k is some constant.

The total energy of the body will then be given by the equation

$$E = AV\rho^{3/2}\chi^{3/2}\left\{B\left(\frac{2(1+\sigma)}{3(1-2\sigma)}\right)^{3/2} + \left(\frac{(1+\sigma)}{3(1-\sigma)}\right)^{3/2}\right\}T^4, \tag{28}$$

or differentiating with respect to T we obtain for the heat capacity of any elastic isotropic body,

$$C = 4AV\rho^{3/2}\chi^{3/2}\left\{B\left(\frac{2(1+\sigma)}{3(1-2\sigma)}\right)^{3/2} + \left(\frac{(1+\sigma)}{3(1-\sigma)}\right)^{3/2}\right\}T^3. \tag{29}$$

A and B are numerical constants which are the same for all materials and their values may be determined once for all by measurements on two substances.

We have thus derived from the principle of similitude and the theory of elasticity not only the important fact that the *heat capacity of an elastic substance is proportional to the third power of the temperature* but also the complete form of the equation connecting heat capacity with temperature and elastic properties.

PART IV. COMPARISON WITH THE EQUATION OF DEBYE.

It can be shown that the above equation agrees with the form taken at low temperatures by Debye's equation for the specific heat of monatomic elastic substances. Debye's equation No. 13[2] reads

$$C/C_\infty = 77.938(T^3/\theta^3).$$

Substituting for θ the value given by Debye's equations (7), (3[1]) and (2), and for C_∞ the value $3Nk$, we may rewrite his formula in the form

$$C = 326\frac{k^4}{h^3}V\rho^{3\,2}\chi^{3/2}\left\{2\left(\frac{2(1+\sigma)}{3(1-2\sigma)}\right)^{3/2} + \left(\frac{(1+\sigma)}{3(1-\sigma)}\right)^{3/2}\right\}T^3,$$

which agrees in form completely with our equation No. 29.

[1] Since ρ and χ occur to the same power, T has to enter to the fourth power in order that the x's cancel.

[2] Loc. cit.

The undetermined factor B which occurs in our equation is seen to have the numerical value 2 in Debye's formula. From our derivation it is evident that B is the ratio of the total energy associated with transverse waves to that associated with longitudinal waves. Our considerations however, were not sufficient for a theoretical determination of the magnitude of this ratio.

The factor $326(k^4/h^3)$ in Debye's formula corresponds with our undetermined numerical factor $4A$. k and h in this factor have the usual significance; k is the so-called gas constant for a single molecule, and h is Planck's Wirkungsquantum.

Debye's calculations have led to a definite numerical value of A since he has made the definite assumption that the energy associated with a mode of vibration of frequency ν is

$$\frac{h\nu}{\epsilon^{\frac{h\nu}{kT}} - 1}.$$

We, on the other hand, have merely assumed that the energy associated with each loop of a stationary wave is *some* function of frequency and temperature, and then determined, except for a constant numerical factor, what this function must be (see equation 27).

With regard to the temperature range over which we can expect our formula for specific heat to hold, we can merely say that at low temperatures where the amplitudes of wave motions are small we may expect *actual substances* to behave like homogeneous media, and hence to obey this formula.[1] For comparison with experimental data we may refer to the article of Debye[2] or to an article by Nernst and Lindemann.[3] We may call attention to the fact that from our derivation we shall expect the same formula to apply to all substances and not merely to monatomic substances as Debye's derivation suggested. As a matter of fact this appears to be true in the case of the measurements of Nernst and Lindemann[4] on potassium and sodium chlorides.

At moderately high temperatures, where there will be a freer motion of the individual atoms, we shall no longer expect that solid bodies will behave like homogeneous media, and hence shall expect increasing deviations from the formula which we have dervied. According to Debye,

[1] The possibility that actual substances behave like continua at very low temperatures was first suggested to the writer by Professor G. N. Lewis. This idea also forms the basis of the considerations of Debye and of Born and Karman, Phys. Zeitschrift, 13, p. 297 (1912). In the case of anisotropic substances we might provisionally take an average value of σ.

[2] Loc. cit.

[3] Sitzungsber. Preuss. Akad., p. 1160 (1912).

[4] Loc. cit.

however, we may expect the specific heat to be proportional to the third power of the temperature within one per cent. up to temperatures as high as $T = \theta/12$ where θ varies for different substances from 72 to 467.[1]

At high temperatures we know that the specific heat of solid substances will obey the law of Dulong and Petit (or of Kopp in the case of polyatomic substances). Hence we might devise as Debye has done a formula which would give a smooth connection between the two known ends of the specific heat curve. For the present, however, we cannot see any satisfactory theoretical derivation for this intermediate part of the curve.

UNIVERSITY OF CALIFORNIA,
BERKELEY, CAL., U. S. A.
May 12, 1914.

[1] Loc. cit., pp. 802 and 817.

NEW BOOKS.

X Rays. By G. W. C. KAYE. New York: Longmans, Green & Co., 1914. Pp. xix + 252. Price $1.25 net.

This little volume is a welcome addition to the literature of this interesting subject. It covers much the same field as Pohl's book but the treatment is less theoretical. The first seven chapters deal with cathode ray phenomena and some features of high tension generators and x-ray tubes, this is followed by about twenty pages devoted to measurement. The treatment of secondary radiation and the brief discussion of interference phenomena in crystals will be very useful to those wishing a general idea of these recent developments but who lack the time required to read the original papers. Appendices contain a brief description of the Coolidge tube and the production of high vacua. The tables and diagrams are well selected and many references are included. All mathematical theory has been omitted but the resumé of experimental data is as complete as the size of the volume permits.

J. S. S.

Molekularkinetik und Molarassociation als Physikochemische Grundvorstellungen. By DR. CARL DRUCKER. Leipzig: Akademische Verlagsgesellschaft, 1913. Pp. 1 + 33.

Dr. Drucker in this "Antrittsvorlesung" emphasizes the importance of the molecular association theory as a working hypothesis for the explanation of a large number of physico-chemical problems. He shows that it should be regarded as a fundamental hypothesis, parallel to and in some cases superior to that of molecular kinetics. This theory is characterized by the assumption of the existence of various molecular complexes, each having definite properties, and of the existence of an equilibrium between them, depending upon the imposed physical conditions, such as temperature and pressure.

The pamphlet gives merely an outline of the theory and a prophesy of its future importance without specific quantitative applications. However, the author has added a large number of references to investigations in which the theory has evidently been used with success.

K. E. G.

Recueil de Constantes Physiques. By HENRI ABRAHAM AND PAUL SACERDOTE. Paris: Gauthier-Villars, 1913. Pp. xvi + 751.

This volume supplements in an admirable manner the standard collections of physical constants. It brings together a great variety of data otherwise not easily found. The tables of the ordinary constants such as density, boiling

point, fusion, specific heat, etc., are augmented by the inclusion of much new material, and a great variety of useful information not found in previous compilations has been added. By the free use of diagrams many relations, which are not conveniently expressed in tabular form, are rendered available. This method is particularly useful in describing the properties of alloys, mixtures, solutions, etc. All the essential formulae and relations are given with an explicit statement of the notation employed. The volume is well indexed and lists of the authors from whom data have been taken accompany the various tables.

E. L. N.

Die Hydrierung durch Katalyse. By PAUL SABATIER. Leipzig: Akademische Verlagsgesellschaft, 1913. Pp. 1 + 20.

This is an address given by Professor Sabatier when he received the Nobel prize in December, 1912. It is a nontechnical description of his general method of producing direct hydration by catalysis and his chemical theory of catalysis. It also contains the history of its conception and development, and the principal applications of the method.

K. E. G.

Dialogues Concerning Two New Sciences. By GALILEO GALILEI. Translated from the Italian and Latin into English by HENRY CREW and ALFONSO DE SALVIO. New York: The Macmillan Company, 1914. Pp. xiii + 300. Price, $2.00 net.

To anyone who is interested in the history of physics or who wishes to send his students to the sources of that science this book will be of great value. When the principal interest of a book is in the ideas which it contains, one can always tolerate a translation, particularly when, as in this instance, the translator has caught the style and spirit of the original. Everyone knows the principal contributions which Galileo made to the science of motion, but most of us do not know how many other notions, which are now the common property of mathematicians and physicists, of less fundamental importance, perhaps, than these, but still of great interest, were first presented or acutely discussed by that great genius. An examination of these dialogues on the strength of materials and on motion will reveal, on almost every page, something to attract attention.

The style of the translators is easy, fluent, and clear. The book is beautifully printed in an antique form and type. The diagrams are reproductions of the originals.

W. F. M.

Die Photographie. By OTTO PRELINGER. Leipzig: B. G. Teubner, 1914. Pp. iv + 110.

Die Luftfahrt. By DR. RAIMUND NIMFÜHR. Leipzig: B. G. Teubner, 1913. Pp. viii + 132.

Unsere Kohlen. By PAUL KUKUK. Leipzig: B. G. Teubner, 1913. Pp. ix + 120.

These booklets are three volumes of Teubner's series, entitled "Aus Natur und Geisteswelt," which is published for the distinct purpose of counteracting the modern tendency towards overspecialization. Each volume is written by an authority on the subject, but in accordance with the aim of the whole series the treatment is elementary and nontechnical. As means for the diffusion of knowledge the booklets certainly accomplish their purpose.

Dr. Prelinger's book presents in a popular form—in addition to a description of the various processes employed in photography and the physical and chemical principles underlying them—a short historical introduction and a condensed chapter on photography in natural colors.

One half of the volume, entitled "Die Luftfahrt" contains a general presentation of the physics of the atmosphere and the fundamental laws of aerostatics and aerodynamics. The second half gives an account of the technical development of balloons and aeroplanes.

"Unsere Kohlen" is written mainly from a geological and geographic point of view. It contains mere references to the physics and chemistry of coal and to the technical side of the subject. Only a few pages are devoted to the occurrence of coal in countries outside of Germany.

K. E. G.

Natural Sources of Energy. By A. H. GIBSON. New York: G. P. Putnam's Sons, 1913. Pp. vi + 131.

This is an interesting little book on the world's energy problem discussing the present and future available sources of energy. It concludes with the consolingly optimistic view that the gradual depletion of the supply of fossil fuel will be more than compensated for by bringing into use the perennial supply of solar energy in its various available forms.

C. A. S.

Electrical and Magnetic Calculations. By A. A. ATKINSON. New York: D. Van Nostrand Company, 1913. Pp. vii + 310. Price, $1.50.

The book, a fourth edition, contains a large number of problems in elementary magnetism and electricity and in the applications of these subjects to electrical engineering. Each chapter contains a short introduction explaining the terms and principles to be used, some illustrative problems with the solutions given in detail, and finally a large number of original problems. These are well selected and contain many valuable practical hints. A number of tables of magnetic and electrical constants are scattered through the book.

As a collection of useful problems the book is of value to teachers in technical high schools or small colleges where only the rudiments of electrical engineering are taught. As a text, however, the book is obsolete. Many of the units described are out of date and loose definitions and statements are frequently

found. The "metre des Archives" and the corresponding kilogram still figure as standards of length and of mass though long ago replaced by the international standards. Similarly the electrical units given in the book are still those proposed by the Chicago conference twenty years ago. "g" is called the force of gravity and a statement like this: "One volt delivering one ampere of current per second represents a power of one watt" is meaningless. Even an elementary text should not lag so many years behind modern knowledge and practice. This is especially to be regretted when it occurs in an otherwise very useful book.

K. E. G.

Modern Seismology. By G. W. WALKER. New York: Longmans, Green and Company, 1913. Pp. xii + 88. Price, $1.40.

The author of this book is a practical seismologist. He gives in it an outline of the theory of seismographs including a study of the conditions of sensitiveness, and of the methods employing in registration; a description of the types of seismographs in present use and directions for their installation. In a second part, he considers the theory of a solid isotropic earth and the application of the seismographic records to the study of particular earthquakes as well as to the testing of theories of the earth's structure. The book contains a number of plates giving specimens of records which illustrate special phenomena and the peculiarities of particular instruments. W. F. M.

Investigacion Experimental Sobre Las Corrientes de Turbulencia en Tubos Capilares. By TRIFON UGARTE. Buenos Ayres, 1913. Pp. 1 + 89.

This extensive thesis contains a full discussion of the formulas which have been proposed to represent the efflux of liquids from capillary tubes as a function of the pressure, the time and the temperature, and of the observations upon which they are based. The author presents an elaborate series of experiments upon the same subject, and verifies the types of these formulas, finding in certain cases that the constants of the formulas are of unexpected simplicity and of general interest.

W. F. M.

Magnetooptische Untersuchungen mitbesonderer Berucksichtigung der magnetischen Zerlegung der Spektrallinien. By P. ZEEMAN. Pp. xi + 242. Price, Mk 9.

This is an exact translation of the original English edition of the author's work, Researches in Magneto-Optics, MacMillan and Co., London, 1913, by Max Iklé. C. A. S.

Die realistische Weltansicht und die Lehre vom Raume. By E. STUDY. Braunschweig: Vieweg and Sohn, 1914. Pp. ix + 145. Price, Mk 4.50.

This book, by a distinguished mathematician, will be found interesting by anyone who is philosophically inclined. The author assumes the realistic

position, and criticizes acutely and with humor the opposing positions of idealism, positivism, and pragmatism. Proceeding to consider the problem of space, he accepts as possible hypotheses for the geometry of space, the Euclidean and the various forms of the non-Euclidean geometry. He accepts as a definition of a point that it is a system of n related numbers, and proceeds from this and similar definitions to describe these geometries by equations. The book ends with a discussion of the part played by the axioms in geometry.

W. F. M.

The Norwegian Aurora Polaris Expedition, 1902–1903. Vol. I. By KR. BIRKELAND. New York: Longmans, Green and Co., 1913. Price, $10.00.

This great volume contains the results of more than ten years' hard and continuous study of magnetic storms and of the origin of terrestrial magnetism. The general thesis which is supported by the author is that magnetic storms are caused by cathode rays projected from the sun. The mathematical theory of the action of such rays, when they are deflected by the magnetic field of the earth, is worked out, and illustrated by the records of magnetic storms. The consequences of theory are also illustrated by a model earth, or terella, past which, when it is magnetized, cathode rays are sent; and the terella is also made a source of cathode rays, and other phenomena, analogous to the zodiacal light, Saturn's rings, and spiral nebulæ are observed. The observations with the terella will be of special interest to the physicist.

W. F. M.